

***The Modal-Hamiltonian Interpretation:
measurement, invariance and ontology***

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CONICET – Universidad de Buenos Aires

**Workshop “Identity, indistinguishability and non-locality
in quantum physics”**

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Systems Postulate: A *quantum system* S is a pair (\mathcal{O}, H) such that

- (i) \mathcal{O} is a space of self-adjoint operators on a Hilbert space \mathcal{H} , representing the *observables* of the system,
- (ii) $H \in \mathcal{O}$ is the time-independent *Hamiltonian* of the system, and
- (iii) if $\rho_0 \in \mathcal{O}'$ (where \mathcal{O}' is the dual space of \mathcal{O}) is the *initial state* of S , ρ_0 evolves according to the Schrödinger (von Neumann) equation

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Pragmatic justification (not needed for chemists)

MHI: Measurement \rightarrow Formal model

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→ The MHI distinguishes between:

- **Reliable** frequency measurement (the state's coefficients can be approximately recovered).
- **Non-reliable** frequency measurement.

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Measurement as a *symmetry breaking* process

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Free particle

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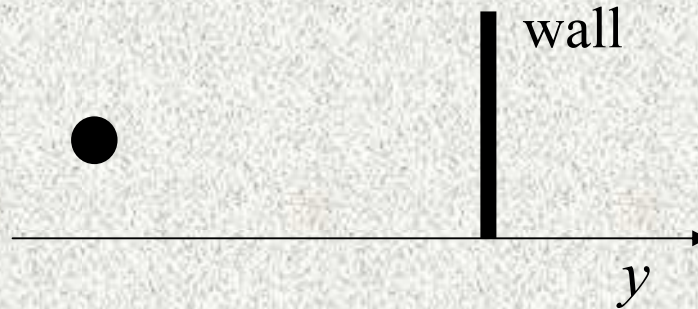
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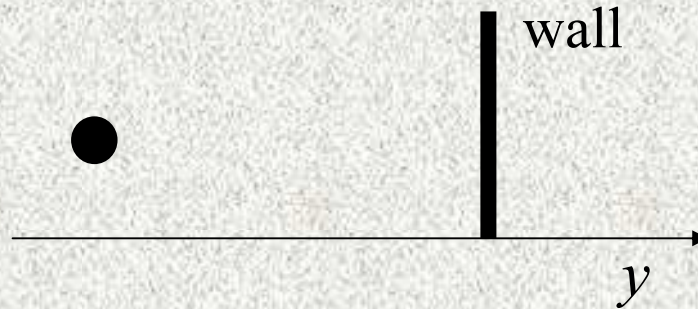
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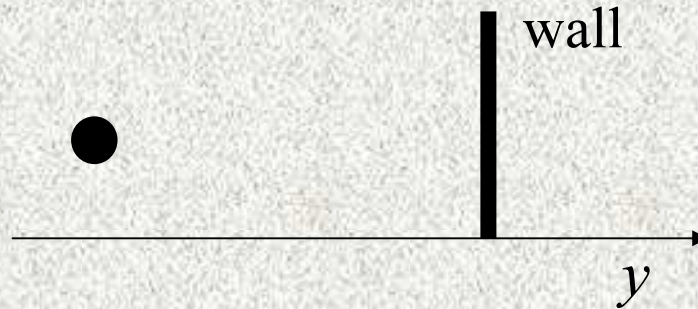
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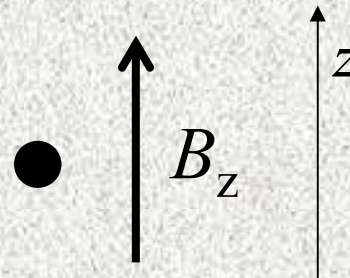
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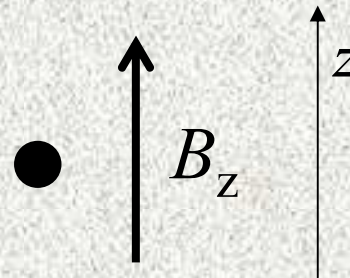
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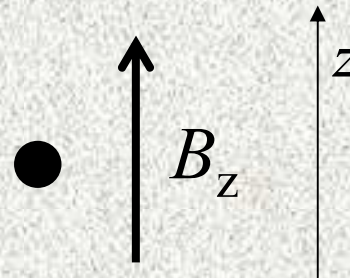
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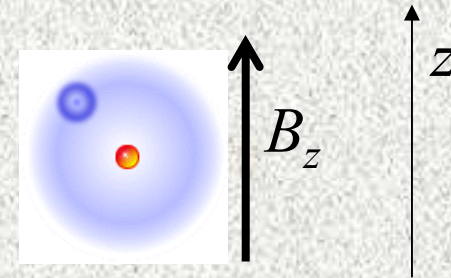
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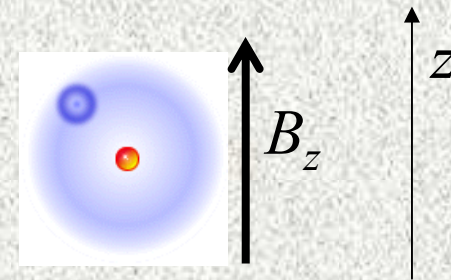
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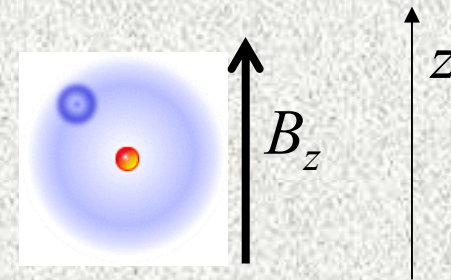
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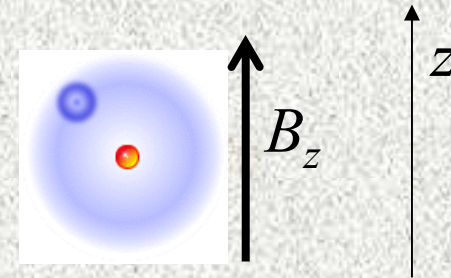
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$$H' = H + \frac{e}{2m_e} B_z L \quad \longrightarrow \quad E_{nlm_l} = E_{nl} \pm \frac{e\hbar |B_z|}{2m_e c} m_l$$

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Measurement is a process that *breaks the symmetry of the Hamiltonian* and, then, turns an otherwise non definite-valued observable into a definite-valued and empirically accessible observable.

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Measurement as a *symmetry breaking* process

It has been successfully applied to several well-known physical models:

- Free-particle
- Free-particle with spin
- Harmonic oscillator
- Free Hydrogen atom
- Zeeman effect
- Fine structure
- The Born-Oppenheimer approximation
- Optical isomerism

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Casimir-Modal interpretation

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QED $\rightarrow M, S^2$ and Q (including internal gauge-symmetries)

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<i>Mathematics</i>	<i>Physics</i>	<i>Ontology</i>
self-adjoint operator A	observable A	instance $[A^i]$ of a universal type-property $[A]$
eigenvalues a_i of A	values of A	possible case-properties $[a_j^i]$ of $[A^i]$
not represented	not represented	actual case-properties $[a_j^i]$ of $[A^i]$
Hilbert space	quantum system	bundle b^i of instances
probability function	probability	propensity
functional ρ	state ρ	codification of propensities

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$$\rho = \rho^S + \rho^A \left\{ \begin{array}{l} \bullet \rho^S = [\rho_{ijmn}^S] \rightarrow \rho_{ijmn}^S = \rho_{mnij}^S \\ \bullet \rho^A = [\rho_{ijmn}^A] \rightarrow \rho_{ijmn}^A = -\rho_{mnij}^A \end{array} \right.$$

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$$\rho = \rho^S + \rho^A \left\{ \begin{array}{l} \bullet \rho^S = [\rho_{ijmn}^S] \rightarrow \rho_{ijmn}^S = \rho_{mnij}^S \\ \bullet \rho^A = [\rho_{ijmn}^A] \rightarrow \rho_{ijmn}^A = -\rho_{mnij}^A \end{array} \right.$$

$$\langle C_k \rangle_\rho = Tr(\rho C_k) = Tr\left[(\rho^S + \rho^A) C_k\right] = Tr(\rho^S C_k) = \langle C_k \rangle_{\rho^S}$$

MHI: Ontology

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The symmetry of the operators representing physical states of aggregates of “elemental particles” is not the result of an ad hoc symmetrization, but is due to ontological reasons, that is, the very structure of the ontology

Thank you!!