Multiple realizability: comparing classical irreversibility and decoherence

Olimpia Lombardi – Sebastian Fortin

CONICET – Universidad de Buenos Aires

Workshop “Multiple Realizability, Causation and Reductive Explanation in Science”

Valparaíso, March 6-7, 2018

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

4.- Coarse graining and partial trace as projections

5.- Coarse description as projection

6.- Coming back to multiple realizability

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

4.- Coarse graining and partial trace as projections

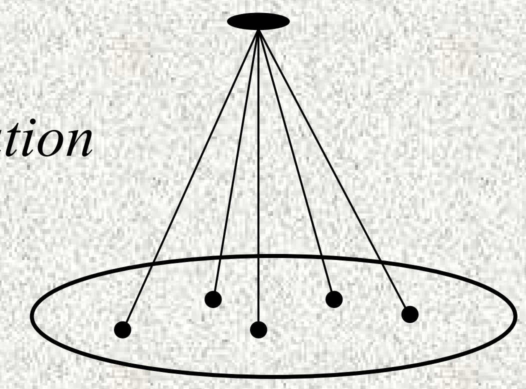
5.- Coarse description as projection

6.- Coming back to multiple realizability

1.- Multiple realizability as a coarse description

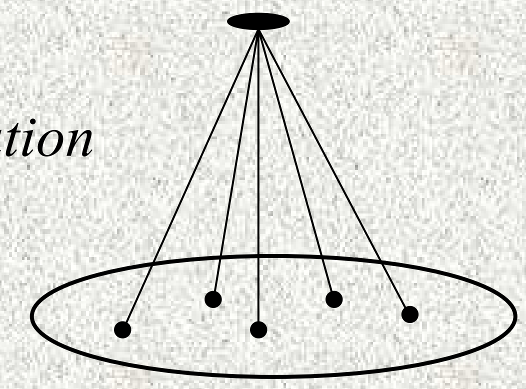
Multiple realizability: supervenience

many-to-one relation



1.- Multiple realizability as a coarse description

Multiple realizability: supervenience

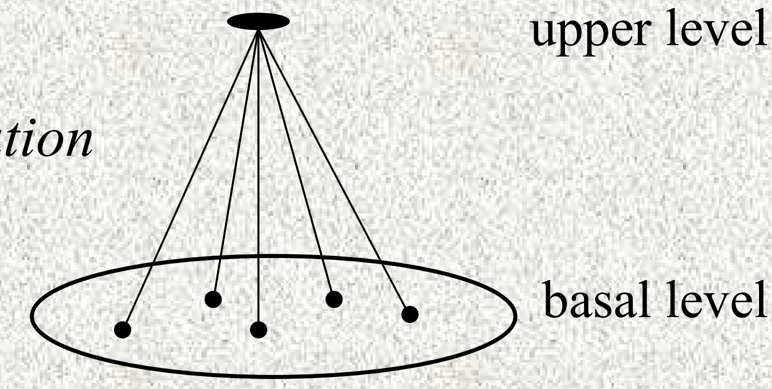


- origins: philosophy of mind
  (excitation of fibers → pain)

1.- Multiple realizability as a coarse description

Multiple realizability: supervenience

many-to-one relation

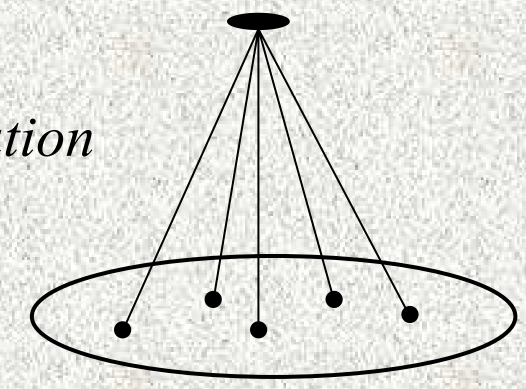


- origins: philosophy of mind
  (excitation of fibers → pain)

- physics: foundations of CSM
  (mechanical states → thermodynamic states)

1.- Multiple realizability as a coarse description

Multiple realizability: supervenience



- origins: philosophy of mind
  (excitation of fibers $\rightarrow$ pain)

- physics: foundations of CSM
  (mechanical states $\rightarrow$ thermodynamic states)

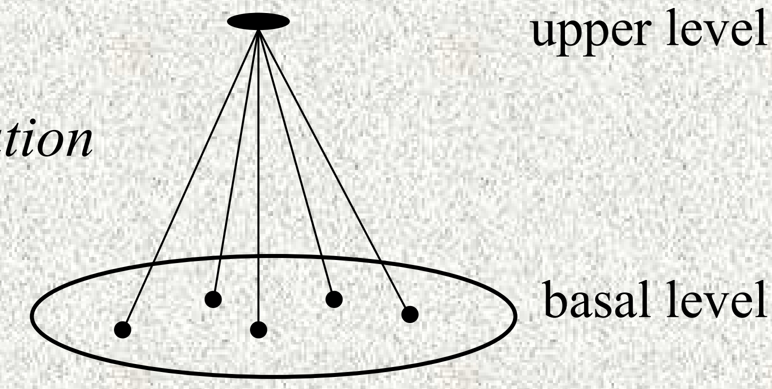
Discussion:
multiple realizability

- $\Rightarrow$ emergence (non-reduction) (Putnam 1975)
- $\not\Rightarrow$ emergence (case of reduction) (Humphreys 1997)

1.- Multiple realizability as a coarse description

Multiple realizability: supervenience

many-to-one relation



- origins: philosophy of mind
  (excitation of fibers → pain)

- physics: foundations of CSM
  (mechanical states → thermodynamic states)

~~Discussion:~~
~~multiple realizability~~

$\Rightarrow$ emergence (non-reduction)
(Putnam 1975)

$\not\Rightarrow$ emergence (case of reduction)
(Humphreys 1997)

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

4.- Coarse graining and partial trace as projections

5.- Coarse description as projection

6.- Coming back to multiple realizability

2.- Coarse description in CSM: coarse graining

Gibbs approach

2.- Coarse description in CSM: coarse graining

Gibbs approach ➡ system of N particles

2.- Coarse description in CSM: coarse graining

Gibbs approach ➡ system of N particles ⤵

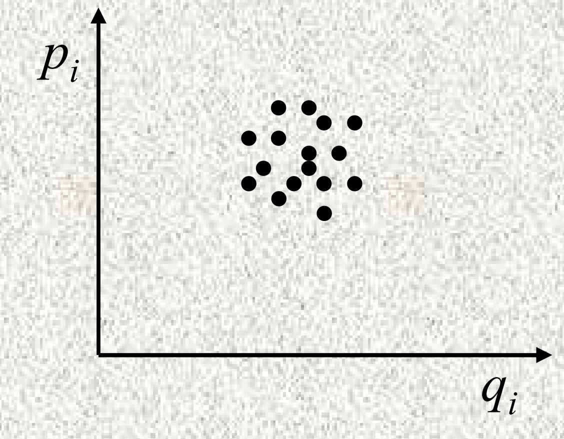
behavior of the representative ensemble

2.- Coarse description in CSM: coarse graining

Gibbs approach ➡ system of N particles ↴

behavior of the representative ensemble

Phase space Γ (6N dimensions)

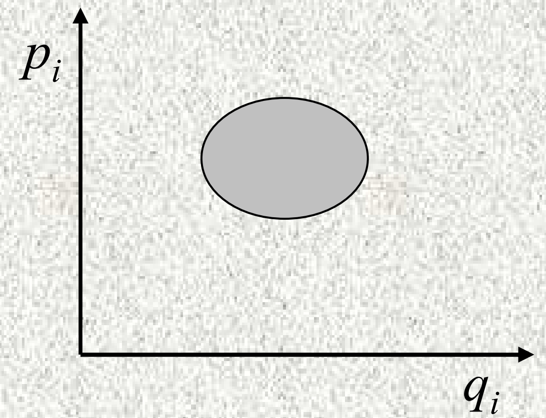


2.- Coarse description in CSM: coarse graining

Gibbs approach ➡ system of N particles ↴

behavior of the representative ensemble

Phase space Γ (6N dimensions)



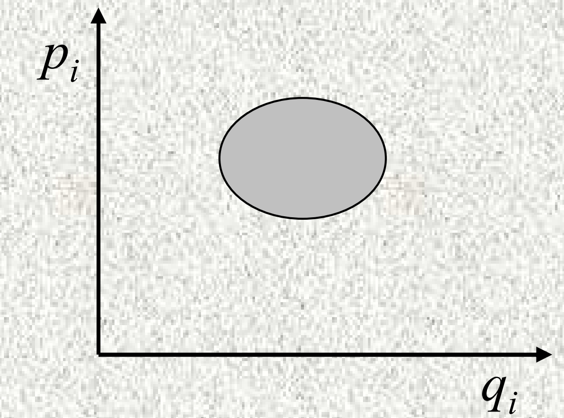
$N \to \infty$: cloud → region

2.- Coarse description in CSM: coarse graining

Gibbs approach ➡ system of N particles ↴

behavior of the representative ensemble

Phase space Γ ($6N$ dimensions)



$N \to \infty$: cloud $\to$ region

$\rho(q_i, p_i, t)$: ***distribution function***

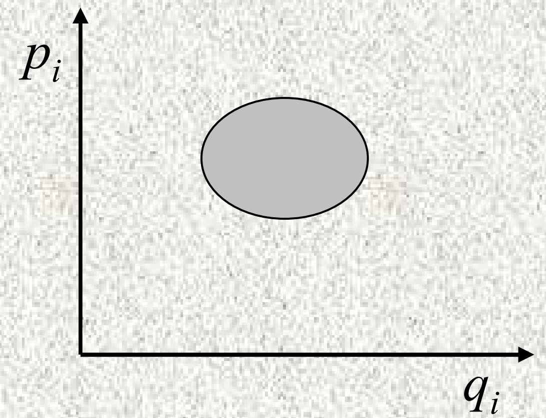
2.- Coarse description in CSM: coarse graining

Gibbs approach ➡ system of N particles ↴

behavior of the representative ensemble

Phase space Γ ($6N$ dimensions)

$N \rightarrow \infty$: cloud → region



$\rho(q_i, p_i, t)$: ***distribution function***

$$\int \ldots \int \rho(q_i, p_i, t)\, dq_i\, dp_i = 1$$

normalized

2.- Coarse description in CSM: coarse graining

$$\rho(q_i, p_i) = \rho(x), \quad x \in \Gamma$$

2.- Coarse description in CSM: coarse graining

$$\rho(q_i, p_i) = \rho(x), \quad x \in \Gamma$$

- Mechanical magnitude: $O : \Gamma \to \mathbb{R}, \quad O \in \mathcal{O}$

phase average $\langle O(x) \rangle_{\rho(x,t)} = \int_\Gamma \rho(x,t)\, O(x)\; dx$

2.- Coarse description in CSM: coarse graining

$$\rho(q_i, p_i) = \rho(x), \quad x \in \Gamma$$

- Mechanical magnitude: $O : \Gamma \to \mathbb{R}, \quad O \in \mathcal{O}$

  phase average $\left\langle O(x) \right\rangle_{\rho(x,t)} = \int_\Gamma \rho(x,t)\, O(x)\; dx$

- Entropy:

  Gibbs entropy $S_G(t) = -k \int_\Gamma \rho(x,t)\, \log \rho(x,t)\; dx$

2.- Coarse description in CSM: coarse graining

$$\rho(q_i, p_i) = \rho(x), \quad x \in \Gamma$$

- Mechanical magnitude: $O : \Gamma \to \mathbb{R}, \quad O \in \mathcal{O}$

phase average $\left\langle O(x) \right\rangle_{\rho(x,t)} = \int_\Gamma \rho(x,t)\, O(x)\, dx$

- Entropy:

Gibbs entropy $S_G(t) = -k \int_\Gamma \rho(x,t) \log \rho(x,t)\, dx$

- Statistical equilibrium: accessible region $\Gamma_A \subset \Gamma$

microcanonical ensemble $\rho_\mu(x) = \begin{cases} cte & \text{for } x \in \Gamma_A \\ 0 & \text{for } x \notin \Gamma_A \end{cases}$

2.- Coarse description in CSM: coarse graining

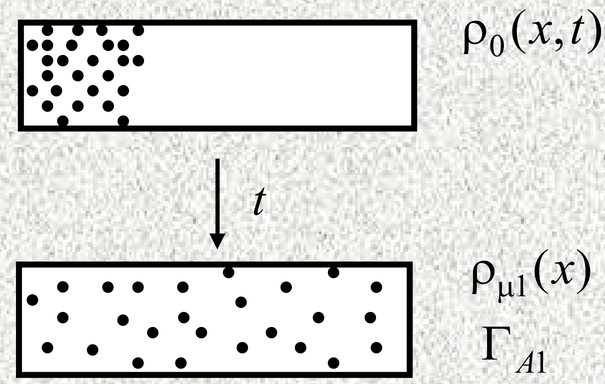
Evolution to equilibrium

2.- Coarse description in CSM: coarse graining

Evolution to equilibrium

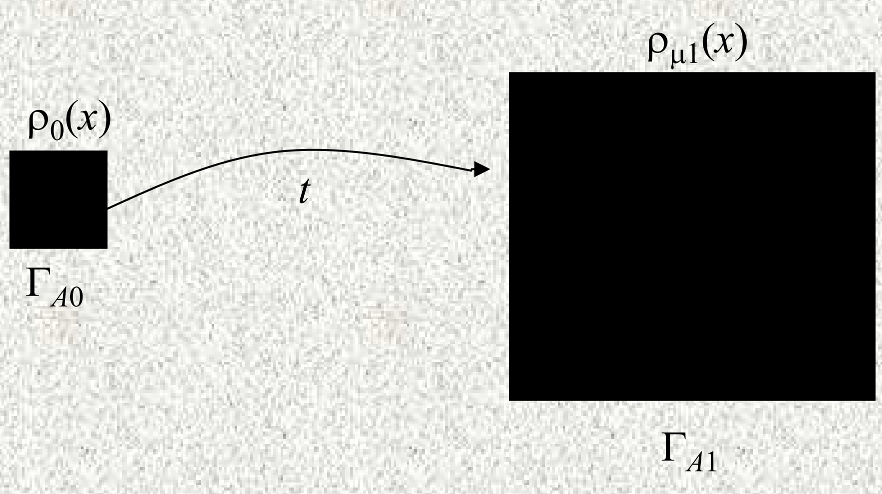
2.- Coarse description in CSM: coarse graining

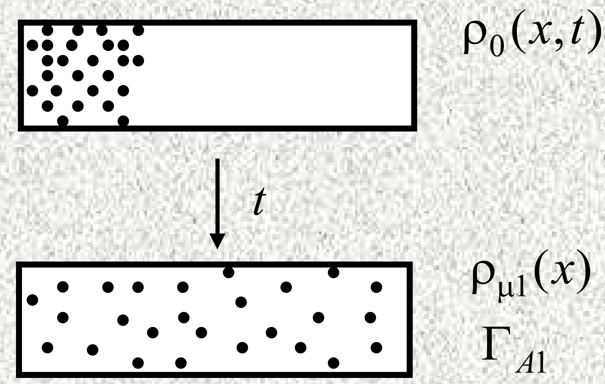
Evolution to equilibrium



2.- Coarse description in CSM: coarse graining

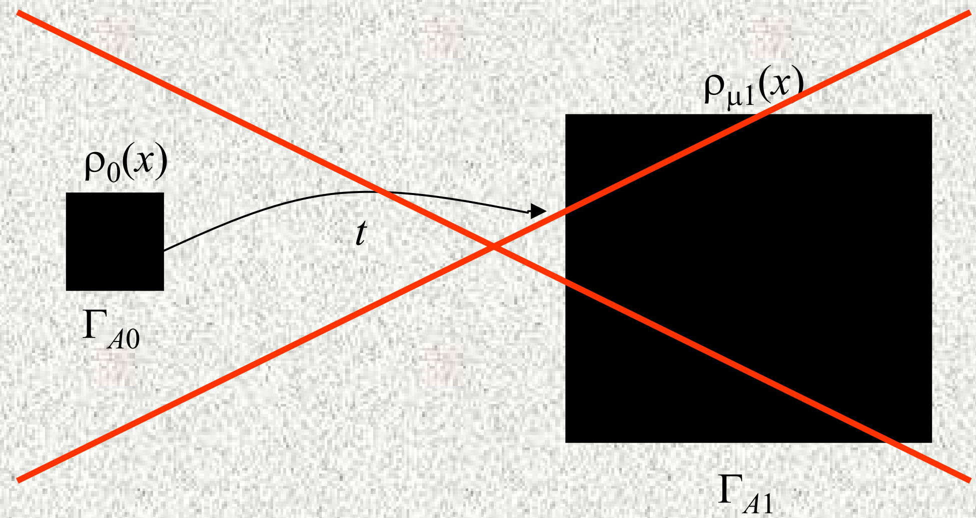
Evolution to equilibrium

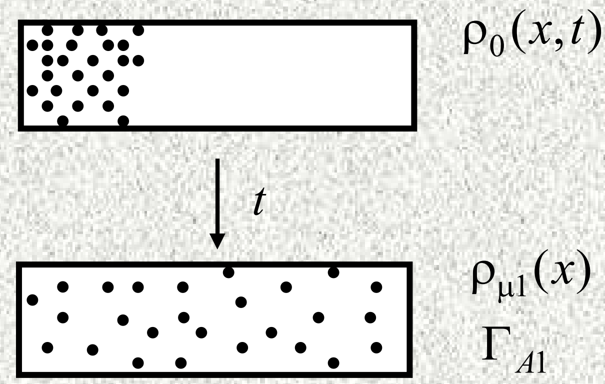




2.- Coarse description in CSM: coarse graining

Evolution to equilibrium

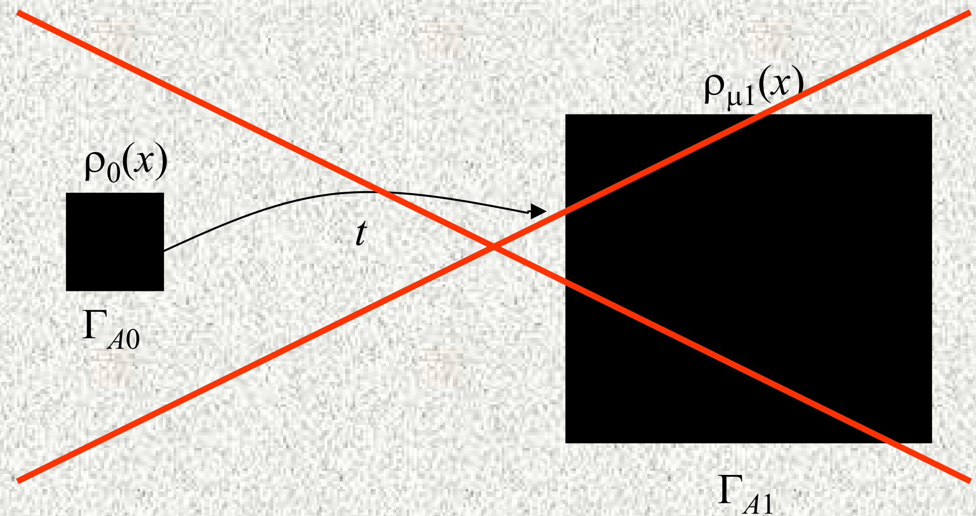


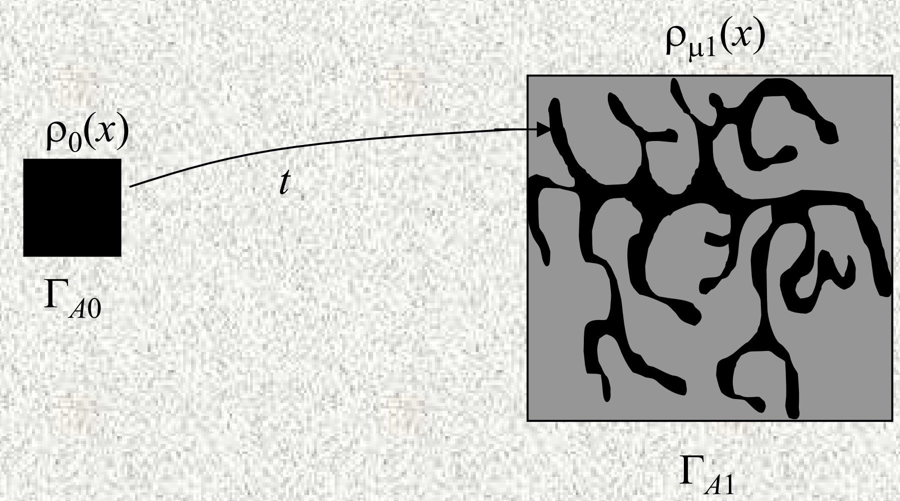


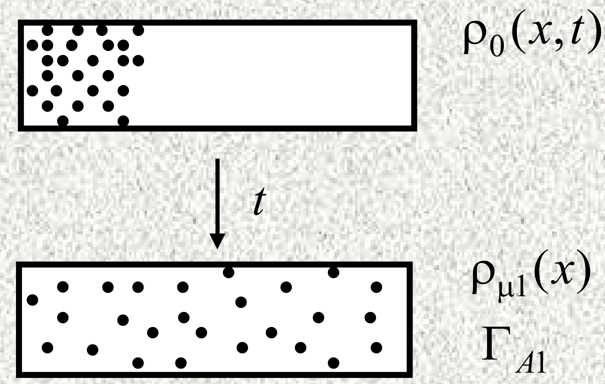
Liouville theorem

2.- Coarse description in CSM: coarse graining

Evolution to equilibrium







Liouville theorem

2.- Coarse description in CSM: coarse graining

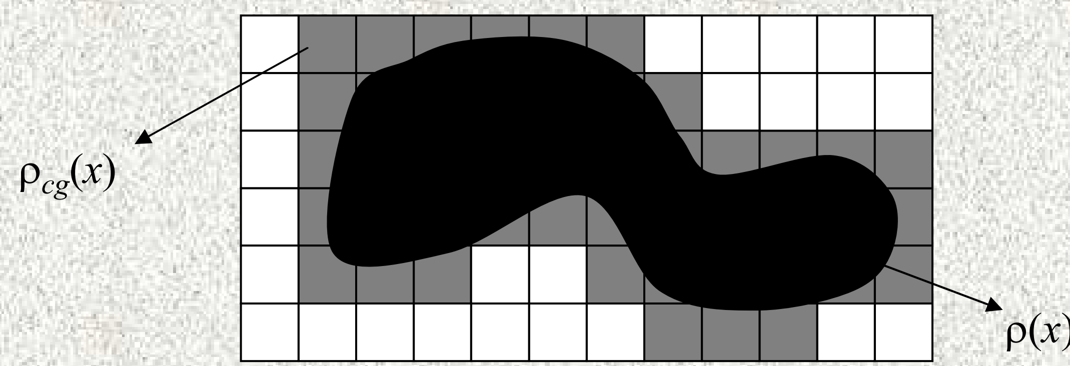
Coarse-grained partition of Γ in cells C_i

$$\rho_{cg}(x) = \begin{cases} \left(1/\mu(C_1)\right)\int_{C_1} \rho(x)\, dx & \text{if } x \in C_1 \\ \left(1/\mu(C_2)\right)\int_{C_2} \rho(x)\, dx & \text{if } x \in C_2 \\ \vdots \end{cases}$$

2.- Coarse description in CSM: coarse graining

Coarse-grained partition of Γ in cells C_i

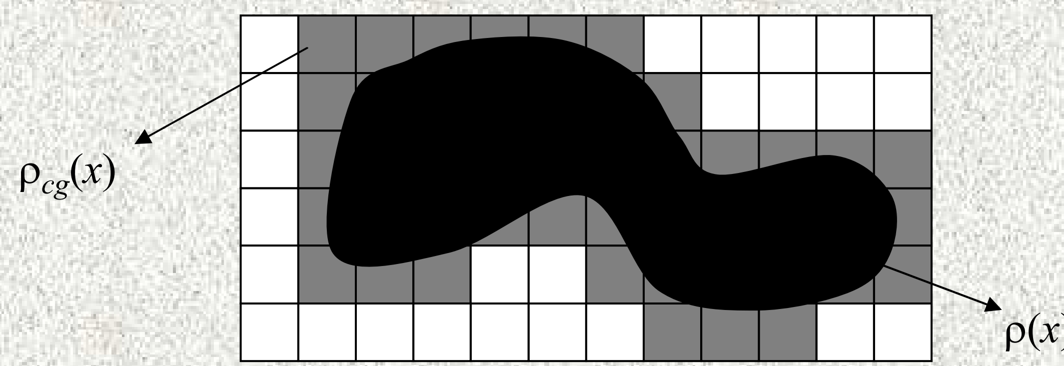
$$\rho_{cg}(x) = \begin{cases} \left(1/\mu(C_1)\right)\int_{C_1} \rho(x)\, dx & \text{if } x \in C_1 \\ \left(1/\mu(C_2)\right)\int_{C_2} \rho(x)\, dx & \text{if } x \in C_2 \\ \vdots \end{cases}$$



2.- Coarse description in CSM: coarse graining

Coarse-grained partition of Γ in cells C_i

$$\rho_{cg}(x) = \begin{cases} \left(1/\mu(C_1)\right)\int_{C_1} \rho(x)\, dx & \text{if } x \in C_1 \\ \left(1/\mu(C_2)\right)\int_{C_2} \rho(x)\, dx & \text{if } x \in C_2 \\ \vdots \end{cases}$$



Multiple realizability
- $\rho(x)$: microstates x
- $\rho_{cg}(x)$: many x for each macrostate C_i

2.- Coarse description in CSM: coarse graining

- ***Coarse-grained distribution*** :

$$\forall O_R \in \mathcal{O}, \quad \left\langle O_R(x) \right\rangle_{\rho(x,t)} = \left\langle O_R(x) \right\rangle_{\rho_{cg}(x,t)}$$

- ***Coarse-grained entropy*** :

$$S_{cg}(t) = -k \int_\Gamma \rho_{cg}(x,t) \log \rho_{cg}(x,t)\, dx$$

2.- Coarse description in CSM: coarse graining

- ***Coarse-grained distribution*** :

$$\forall O_R \in \mathcal{O}, \quad \langle O_R(x) \rangle_{\rho(x,t)} = \langle O_R(x) \rangle_{\rho_{cg}(x,t)}$$

- ***Coarse-grained entropy*** :

$$S_{cg}(t) = -k \int_\Gamma \rho_{cg}(x,t) \log \rho_{cg}(x,t) \, dx$$

Mixing :

$$S_{cg}(t) \xrightarrow{t >> t_0} S_{cg(eq)}$$

$$\lim_{t\to\infty} \langle O_R(x) \rangle_{\rho_{cg}(x,t)} = \langle O_R(x) \rangle_{\rho_{cg(eq)}(x)}$$

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

4.- Coarse graining and partial trace as projections

5.- Coarse description as projection

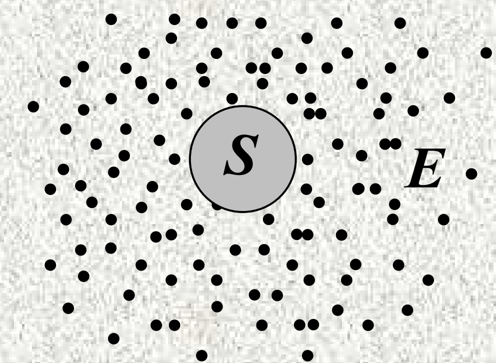
6.- Coming back to multiple realizability

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between S and E

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between *S* and *E*

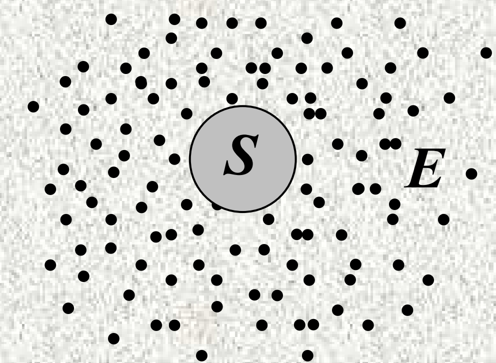


$$\left|\Psi_S(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle$$

$$\left|\Psi_{SE}(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\phi_E(t=0)\right\rangle$$

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between *S* and *E*



$$\left|\Psi_S(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle$$

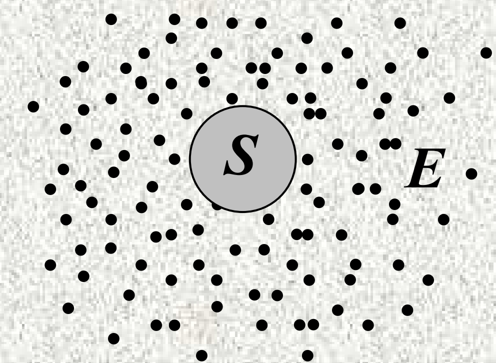
$$\left|\Psi_{SE}(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\phi_E(t=0)\right\rangle$$

$$\Big\downarrow H_{int}$$

$$\left|\Psi_{SE}(t)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\varepsilon_i(t)\right\rangle$$

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between *S* and *E*



$$\left|\Psi_S(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle$$

$$\left|\Psi_{SE}(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\phi_E(t=0)\right\rangle$$

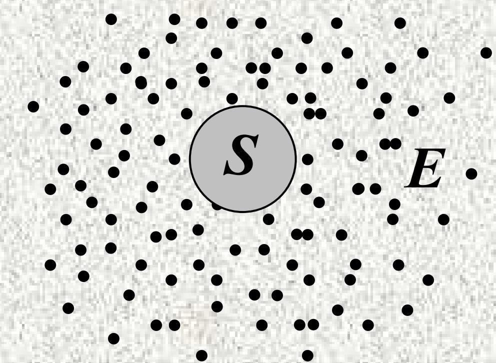
$$\Big\downarrow H_{int}$$

$$\left|\Psi_{SE}(t)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\varepsilon_i(t)\right\rangle$$

$$\hat{\rho}_{SE}(t) = \left|\Psi_{SE}(t)\right\rangle\left\langle\Psi_{SE}(t)\right| = \sum_{ij} c_i c_j^* \left|\varphi_i\right\rangle \otimes \left|\varepsilon_i(t)\right\rangle\left\langle\varphi_j\right| \otimes \left\langle\varepsilon_j(t)\right|$$

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between *S* and *E*



$$\left|\Psi_S(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle$$

$$\left|\Psi_{SE}(t=0)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\phi_E(t=0)\right\rangle$$

$$\Big\downarrow H_{int}$$

$$\left|\Psi_{SE}(t)\right\rangle = \sum_i c_i \left|\varphi_i\right\rangle \otimes \left|\varepsilon_i(t)\right\rangle$$

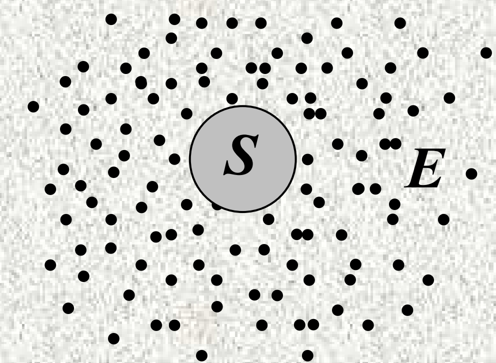
$$\hat{\rho}_{SE}(t) = \left|\Psi_{SE}(t)\right\rangle\left\langle\Psi_{SE}(t)\right| = \sum_{ij} c_i c_j^* \left|\varphi_i\right\rangle \otimes \left|\varepsilon_i(t)\right\rangle \left\langle\varphi_j\right| \otimes \left\langle\varepsilon_j(t)\right|$$

$$\downarrow$$

$c_i c_j^* \neq 0$ with $i \neq j$ $\rightarrow$ quantum correlations

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between S and E

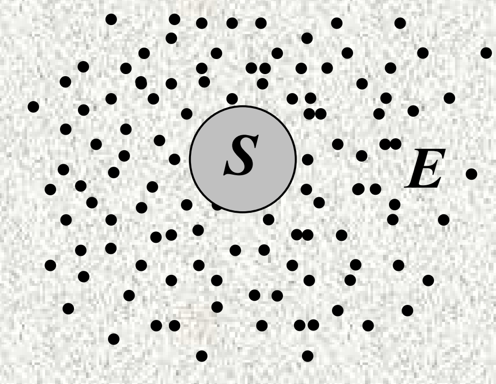


$$\hat{\rho}_r^S(t) = Tr_E\, \hat{\rho}_{SE}(t) = \sum_{ij} \left\langle \varepsilon_i(t) \middle| \hat{\rho}_{SE}(t) \middle| \varepsilon_j(t) \right\rangle$$

→ ***reduced state of S***

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between S and E



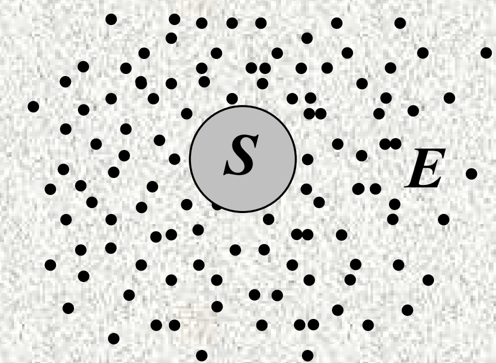
$$\hat{\rho}_r^S(t) = Tr_E\, \hat{\rho}_{SE}(t) = \sum_{ij} \left\langle \varepsilon_i(t) \middle| \hat{\rho}_{SE}(t) \middle| \varepsilon_j(t) \right\rangle$$

→ ***reduced state of S***

$$\hat{\rho}_r^S(t) = \sum_{ij} c_i c_j^* \left\langle \varepsilon_i(t) \middle| \varepsilon_j(t) \right\rangle \left| \varphi_i \right\rangle \left\langle \varphi_j \right|$$

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between *S* and *E*



$$\hat{\rho}_r^S(t) = Tr_E\, \hat{\rho}_{SE}(t) = \sum_{ij} \langle \varepsilon_i(t) | \hat{\rho}_{SE}(t) | \varepsilon_j(t) \rangle$$

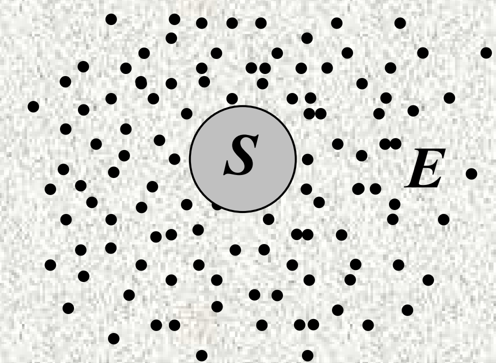
↳ ***reduced state of S***

$$\hat{\rho}_r^S(t) = \sum_{ij} c_i c_j^* \langle \varepsilon_i(t) | \varepsilon_j(t) \rangle | \varphi_i \rangle \langle \varphi_j |$$

$|\varepsilon_i(t)\rangle$ tend to orthogonality: $\langle \varepsilon_i(t) | \varepsilon_j(t) \rangle \xrightarrow{t>>t_D} 0 \quad (i \neq j)$

3.- Coarse description in QM: partial trace

Decoherence: consequence of the interaction between *S* and *E*



$$\hat{\rho}_r^S(t) = Tr_E\,\hat{\rho}_{SE}(t) = \sum_{ij} \left\langle \varepsilon_i(t) \middle| \hat{\rho}_{SE}(t) \middle| \varepsilon_j(t) \right\rangle$$

→ ***reduced state of S***

$$\hat{\rho}_r^S(t) = \sum_{ij} c_i c_j^* \left\langle \varepsilon_i(t) \middle| \varepsilon_j(t) \right\rangle \left| \varphi_i \right\rangle \left\langle \varphi_j \right|$$

$\left| \varepsilon_i(t) \right\rangle$ tend to orthogonality: $\left\langle \varepsilon_i(t) \middle| \varepsilon_j(t) \right\rangle \xrightarrow{t>>t_D} 0 \quad (i \neq j)$

$$\hat{\rho}_r^S(t) \xrightarrow{t>>t_D} \hat{\rho}_r^S = \sum_i \left| c_i \right|^2 \left| \varphi_i \right\rangle \left\langle \varphi_i \right|$$

mixture without quantum correlations

3.- Coarse description in QM: partial trace

Definition of ***reduced state***

$$\hat{O}_S \in \mathcal{H}_S \otimes \mathcal{H}_S \qquad \hat{\rho}(t) \in \mathcal{H}_{SE} \otimes \mathcal{H}_{SE} \quad (\mathcal{H}_{SE} = \mathcal{H}_S \otimes \mathcal{H}_E)$$

$\hat{\rho}_r^S$ is such that: $\forall \hat{O}_R = \hat{O}_S \otimes \hat{I}_E \in \hat{\mathcal{O}} = \mathcal{H}_{SE} \otimes \mathcal{H}_{SE}$

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S(t)}$$

3.- Coarse description in QM: partial trace

Definition of ***reduced state***

$$\hat{O}_S \in \mathcal{H}_S \otimes \mathcal{H}_S \qquad \hat{\rho}(t) \in \mathcal{H}_{SE} \otimes \mathcal{H}_{SE} \quad (\mathcal{H}_{SE} = \mathcal{H}_S \otimes \mathcal{H}_E)$$

$\hat{\rho}_r^S$ is such that: $\forall \hat{O}_R = \hat{O}_S \otimes \hat{I}_E \in \hat{\mathcal{O}} = \mathcal{H}_{SE} \otimes \mathcal{H}_{SE}$

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S(t)}$$

- D'Espagnat: ⟶ $\hat{\rho}_r^S$ is an ***improper mixture***
- Schlosshauer: ⟶ $\hat{\rho}_r^S$ is a ***calculational tool*** for computing expectation values

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

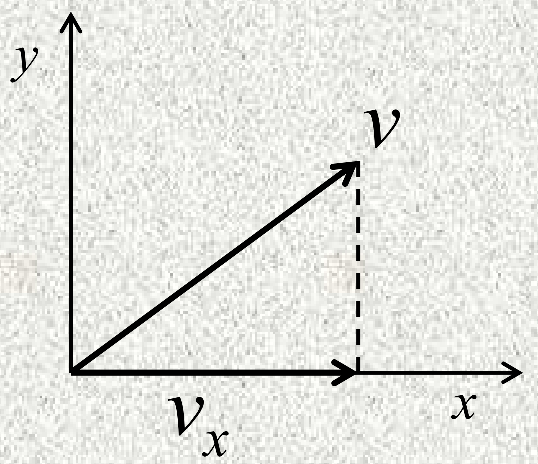
4.- Coarse graining and partial trace as projections

5.- Coarse description as projection

6.- Coming back to multiple realizability

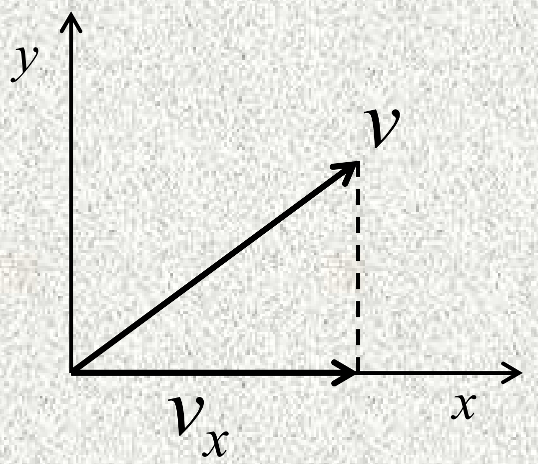
4.- Coarse graining and partial trace as projections

Projector: traditional concept



4.- Coarse graining and partial trace as projections

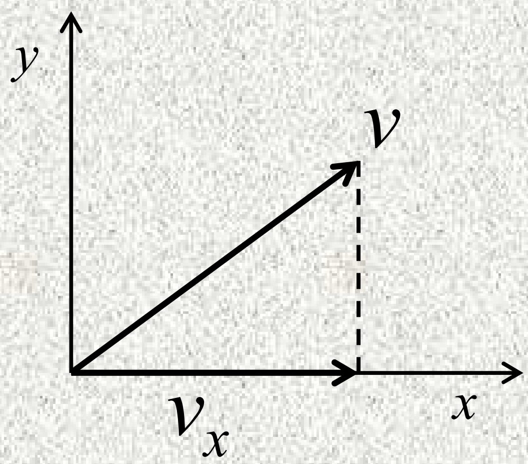
Projector: traditional concept



$$v_x = \Pi\, v$$

4.- Coarse graining and partial trace as projections

Projector: traditional concept

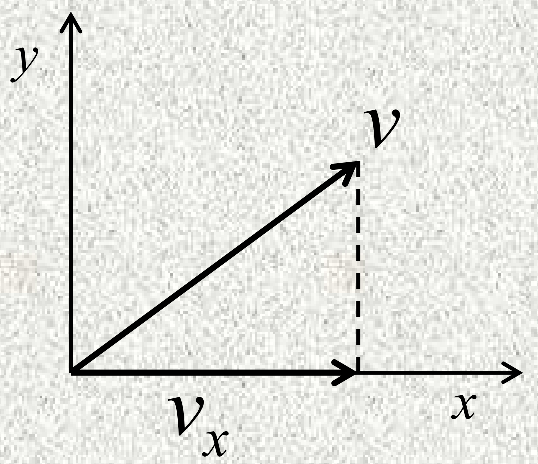


$$v_x = \Pi\, v$$

$$\Pi\, v_x = \Pi\,\Pi\, v = v_x = \Pi\, v$$

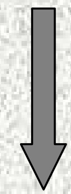
4.- Coarse graining and partial trace as projections

Projector: traditional concept



$$v_x = \Pi\, v$$

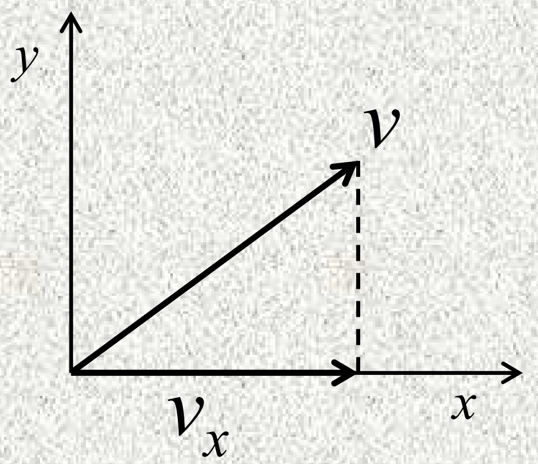
$$\Pi\, v_x = \Pi\,\Pi\, v = v_x = \Pi\, v$$



Generalization

4.- Coarse graining and partial trace as projections

Projector: traditional concept



$$v_x = \Pi \, v$$

$$\Pi \, v_x = \Pi \, \Pi \, v = v_x = \Pi \, v$$

Generalization

Projector: operator Π such that $\Pi \, \Pi = \Pi$

4.- Coarse graining and partial trace as projections

Coarse-graining in CSM as projection

$$\rho_{cg}(x) = \left\{ \frac{1}{\mu(C_i)} \int_{C_i} \rho(x)\, dx \quad \text{if } x \in C_i \right.$$

4.- Coarse graining and partial trace as projections

Coarse-graining in CSM as projection

$$\rho_{cg}(x) = \left\{ \frac{1}{\mu(C_i)} \int_{C_i} \rho(x)\, dx \quad \text{if } x \in C_i \right.$$

$$\rho_{cg}(x) = \Pi\,\rho(x) = \Pi\,\Pi\,\rho(x)$$

Easy to see because $\rho(x)$ and $\rho_{cg}(x)$ have the same dimensionality (both defined on Γ of $6N$ dimensions)

4.- Coarse graining and partial trace as projections

Partial trace in QM as projection (Fortin & Lombardi 2014)

$$\hat{\rho}_r^S = Tr_E \, \hat{\rho}_{SE}$$

Partial trace in QM as projection (Fortin & Lombardi 2014)

$$\hat{\rho}_r^S = Tr_E\, \hat{\rho}_{SE}$$

$$\hookrightarrow \in \mathcal{H}_{SE} \otimes \mathcal{H}_{SE}$$

4.- Coarse graining and partial trace as projections

Partial trace in QM as projection (Fortin & Lombardi 2014)

$$\hat{\rho}_r^S = Tr_E \, \hat{\rho}_{SE}$$

$\hat{\rho}_{SE} \in \mathcal{H}_{SE} \otimes \mathcal{H}_{SE}$

$\hat{\rho}_r^S \in \mathcal{H}_S \otimes \mathcal{H}_S$

4.- Coarse graining and partial trace as projections

Partial trace in QM as projection (Fortin & Lombardi 2014)

$$\hat{\rho}_r^S = Tr_E \, \hat{\rho}_{SE}$$

$\hat{\rho}_{SE} \in \mathcal{H}_{SE} \otimes \mathcal{H}_{SE}$

$\hat{\rho}_r^S \in \mathcal{H}_S \otimes \mathcal{H}_S$

For dimensional reasons:

$$\hat{\rho}_r^S \neq \Pi \, \hat{\rho}_{SE}$$

4.- Coarse graining and partial trace as projections

Partial trace in QM as projection (Fortin & Lombardi 2014)

However

$$\left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S} = \left\langle \hat{O}_S \otimes \hat{I}_E \right\rangle_{\hat{\rho}_{cg}^{SE}}$$

For dimensional reasons:

$$\hat{\rho}_r^S \neq \Pi \, \hat{\rho}_{SE}$$

4.- Coarse graining and partial trace as projections

Partial trace in QM as projection (Fortin & Lombardi 2014)

However

$$\left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S} = \left\langle \hat{O}_S \otimes \hat{I}_E \right\rangle_{\hat{\rho}_{cg}^{SE}}$$

For dimensional reasons:

$$\hat{\rho}_r^S \neq \Pi \, \hat{\rho}_{SE}$$

$\hat{\rho}_{cg}^{SE}$ → can be defined as a projection

4.- Coarse graining and partial trace as projections

Partial trace in QM as projection (Fortin & Lombardi 2014)

For dimensional reasons:

$$\hat{\rho}_r^S \neq \Pi \, \hat{\rho}_{SE}$$

However

$$\left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S} = \left\langle \hat{O}_S \otimes \hat{I}_E \right\rangle_{\hat{\rho}_{cg}^{SE}}$$

$\hat{\rho}_{cg}^{SE}$ → can be defined as a projection

$$\hat{\rho}_{cg}^{SE} = \Pi \, \hat{\rho}_{SE} = \hat{\rho}_r^S \otimes \tilde{\delta}_E$$

Partial trace in QM as projection (Fortin & Lombardi 2014)

For dimensional reasons:

$$\hat{\rho}_r^S \neq \Pi \, \hat{\rho}_{SE}$$

However

$$\left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S} = \left\langle \hat{O}_S \otimes \hat{I}_E \right\rangle_{\hat{\rho}_{cg}^{SE}}$$

$\hat{\rho}_{cg}^{SE}$ → can be defined as a projection

$$\hat{\rho}_{cg}^{SE} = \Pi \, \hat{\rho}_{SE} = \hat{\rho}_r^S \otimes \tilde{\delta}_E$$

$\tilde{\delta}_E$ → normalized identity operator with coefficients:

$$\tilde{\delta}_{E\alpha\beta} = \delta_{\alpha\beta} / \sum_\gamma \delta_{\gamma\gamma}$$

Partial trace in QM as projection (Fortin & Lombardi 2014)

However

For dimensional reasons:

$$\hat{\rho}_r^S \neq \Pi\,\hat{\rho}_{SE}$$

$$\left\langle \hat{O}_S \right\rangle_{\hat{\rho}_r^S} = \left\langle \hat{O}_S \otimes \hat{I}_E \right\rangle_{\hat{\rho}_{cg}^{SE}}$$

$\hat{\rho}_{cg}^{SE}$ → can be defined as a projection

$$\hat{\rho}_{cg}^{SE} = \Pi\,\hat{\rho}_{SE} = \hat{\rho}_r^S \otimes \tilde{\delta}_E$$

$\tilde{\delta}_E$ → normalized identity operator with coefficients:

$$\tilde{\delta}_{E\alpha\beta} = \delta_{\alpha\beta} / \sum_\gamma \delta_{\gamma\gamma}$$

It can be proved that $\hat{\rho}_r^S = Tr_E\,\hat{\rho}_{cg}^{SE}$

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

4.- Coarse graining and partial trace as projections

5.- Coarse description as projection

6.- Coming back to multiple realizability

5.- Coarse description as projection

CSM	**QM**
$\rho(x,t) \quad O(x) \in \mathcal{O}$	$\hat{\rho}(t) \quad \hat{O} \in \hat{\mathcal{O}}$
$\partial\rho / \partial t = -\{H, \rho\}$	$\partial\hat{\rho} / \partial t = -\, {}^{i}\!/\!{}_{\hbar} \left[\hat{H}, \hat{\rho}\right]$
$\left\langle O(x) \right\rangle_{\rho(x,t)} = \int_{\Gamma} \rho(x,t)\, O(x)\; dx$	$\left\langle \hat{O} \right\rangle_{\hat{\rho}(t)} = Tr\left(\hat{\rho}\, \hat{O}\right)$

5.- Coarse description as projection

CSM	QM
$\rho(x,t) \quad O(x) \in \mathcal{O}$	$\hat{\rho}(t) \quad \hat{O} \in \hat{\mathcal{O}}$
$\partial\rho / \partial t = -\{H, \rho\}$	$\partial\hat{\rho} / \partial t = -{}^{i}\!/\!_{\hbar}\left[\hat{H}, \hat{\rho}\right]$
$\left\langle O(x) \right\rangle_{\rho(x,t)} = \int_{\Gamma} \rho(x,t)\, O(x)\, dx$	$\left\langle \hat{O} \right\rangle_{\hat{\rho}(t)} = Tr\left(\hat{\rho}\, \hat{O}\right)$

Let us define a ρ_{CD} such that, for some $O_R \in \mathcal{O}$

$$\left\langle O_R \right\rangle_{\rho} = \left\langle O_R \right\rangle_{\rho_{CD}}$$

5.- Coarse description as projection

CSM	QM
$\rho(x,t) \quad O(x) \in \mathcal{O}$	$\hat{\rho}(t) \quad \hat{O} \in \hat{\mathcal{O}}$
$\partial\rho / \partial t = -\{H, \rho\}$	$\partial\hat{\rho} / \partial t = -{}^{i}\!/_{\hbar}\left[\hat{H}, \hat{\rho}\right]$
$\langle O(x)\rangle_{\rho(x,t)} = \int_{\Gamma} \rho(x,t)\, O(x)\; dx$	$\langle \hat{O} \rangle_{\hat{\rho}(t)} = Tr\left(\hat{\rho}\, \hat{O}\right)$

Let us define a ρ_{CD} such that, for some $O_R \in \mathcal{O}$

$$\langle O_R \rangle_{\rho} = \langle O_R \rangle_{\rho_{CD}}$$

$\rho_{CD} \to \rho_{CD}(x)$	$\rho_{CD} \to \hat{\rho}_{CD}$
$\rho_{CD}(x) = \Pi\, \rho(x) = \rho_{cg}(x)$	$\hat{\rho}_{CD} = \Pi\, \hat{\rho}_{12} = \hat{\rho}_{r}^{1} \otimes \tilde{\delta}_{2} = \hat{\rho}_{cg}^{12}$

5.- Coarse description as projection

CSM	QM
$\rho(x,t) \quad O(x) \in \mathcal{O}$	$\hat{\rho}(t) \quad \hat{O} \in \hat{\mathcal{O}}$
$\partial\rho / \partial t = -\{H, \rho\}$	$\partial\hat{\rho} / \partial t = -\, {}^{i}\!/_{\hbar} \left[\hat{H}, \hat{\rho}\right]$
$\langle O(x) \rangle_{\rho(x,t)} = \int_{\Gamma} \rho(x,t)\, O(x)\; dx$	$\left\langle \hat{O} \right\rangle_{\hat{\rho}(t)} = Tr\left(\hat{\rho}\, \hat{O}\right)$

Let us define a ρ_{CD} such that, for some $O_R \in \mathcal{O}$

$$\langle O_R \rangle_{\rho} = \langle O_R \rangle_{\rho_{CD}}$$

$\rho_{CD} \rightarrow \rho_{CD}(x)$

$$\rho_{CD}(x) = \Pi\, \rho(x) = \rho_{cg}(x)$$

We lose some information about the exact values of all the dynamical variables
(Gross observables)

$\rho_{CD} \rightarrow \hat{\rho}_{CD}$

$$\hat{\rho}_{CD} = \Pi\, \hat{\rho}_{12} = \hat{\rho}_r^1 \otimes \tilde{\delta}_2 = \hat{\rho}_{cg}^{12}$$

We lose all the information about some dynamical variables, those corresponding to a subsystem
(observables of the subsystem)

5.- Coarse description as projection

CSM	**QM**
$\rho(x,t) \quad O(x)\in\mathcal{O}$	$\hat{\rho}(t) \quad \hat{O}\in\hat{\mathcal{O}}$
$\partial\rho/\partial t = -\{H,\rho\}$	$\partial\hat{\rho}/\partial t = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]$
$\langle O(x)\rangle_{\rho(x,t)} = \int_\Gamma \rho(x,t)\,O(x)\,dx$	$\langle \hat{O}\rangle_{\hat{\rho}(t)} = Tr\left(\hat{\rho}\,\hat{O}\right)$

Let us define a ρ_{CD} such that, for some $O_R\in\mathcal{O}$

$$\langle O_R\rangle_\rho = \langle O_R\rangle_{\rho_{CD}}$$

$\rho_{CD}\to\rho_{CD}(x)$

$\rho_{CD}\to\hat{\rho}_{CD}$

$$\rho_{CD}(x) = \Pi\,\rho(x) = \rho_{cg}(x)$$

$$\hat{\rho}_{CD} = \Pi\,\hat{\rho}_{12} = \hat{\rho}_r^1\otimes\tilde{\delta}_2 = \hat{\rho}_{cg}^{12}$$

We lose some information about the exact values of all the dynamical variables
(Gross observables)

We lose all the information about some dynamical variables, those corresponding to a subsystem
(observables of the subsystem)

OUTLINE

1.- Multiple realizability as a coarse description

2.- Coarse description in CSM: coarse graining

3.- Coarse description in QM: partial trace

4.- Coarse graining and partial trace as projections

5.- Coarse description as projection

6.- Coming back to multiple realizability

6.- Coming back to multiple realizability

- Supervenience is not paradigmatically multiple realizability: partial trace is not a simple many-to-one relation

6.- Coming back to multiple realizability

- Supervenience is not paradigmatically multiple realizability: partial trace is not a simple many-to-one relation

- This relationship is not even supervenience:
  There may be a change in the upper level without change in the bottom level (changing the set of relevant observables)

6.- Coming back to multiple realizability

- Supervenience is not paradigmatically multiple realizability:
  partial trace is not a simple many-to-one relation

- This relationship is not even supervenience:
  There may be a change in the upper level without change in the bottom level (changing the set of relevant observables)

- The moral of the story:
  What is relevant is the relationship between an underlying level, represented by states, with what is empirically accessible, represented by observables.
  Instead of supervenience or multiple realizablity, I prefer ***intra-theory emergence***

Thank you!!!