PREFACE

The aim of this book is to supply a comprehensive presentation of a new member of the family of the modal interpretations of quantum mechanics. According to our modal-Hamiltonian interpretation, the Hamiltonian of the quantum system plays a decisive role in the definition of systems and subsystems, and in the rule that selects the observables whose possible values become actual. We shall begin with introducing the main interpretative postulates and by proving their Galilean invariance. Then we shall argue for the physical relevance of the interpretation. We shall also show how our interpretation solves the quantum measurement problem, both in the ideal and in the non-ideal versions, and why this solution is compatible with the environment-induced decoherence approach. Finally, we shall describe, from a philosophical viewpoint, the quantum ontology implied by the interpretation.

Chapter 1

INTRODUCTION

The problem of the interpretation of quantum mechanics is certainly one of the most discussed topics in the foundations of physics. More than a hundred years after the birth of the theory, there is still no consensus about how its main concepts have to be understood. Nevertheless, during the last decades, the traditional instrumentalist interpretations have begun to loose their original strength, and several realist readings have been proposed. Among them, and inspired in the works of van Fraassen (1972, 1973, 1974), modal interpretations are realist, non-collapse interpretations, according to which the quantum state of a system describes the possible properties of the system rather than the properties that it actually possesses.

Since the 1980s, several modal interpretations have been proposed (see Dieks and Vermaas 1998, Dickson and Dieks 2008). They share central features (for a clear summary, see Dieks 2007, Section 1), but they differ to each other in the particular rule of actual-value ascription, that is, the rule that selects the properties having an actual, and not merely possible, value. Recently, we have proposed a new member of the modal "family", the modal-Hamiltonian interpretation (MHI), according to which the Hamiltonian of the system plays a central role both in the definition of quantum systems and subsystems and in the selection of the actual-valued properties (Lombardi and Castagnino 2008, Castagnino and Lombardi 2008, Ardenghi, Castagnino and Lombardi 2009, Lombardi, Castagnino and Ardenghi 2010). The aim of this work is to supply a comprehensive presentation of the MHI, and for this purpose the work is organized as follows. In Chapter 2 the main interpretative postulates are introduced, and in Chapter 3 the Galilean invariance of the interpretation is proved. Then, in Chapter 4 the physical relevance of the

interpretation is argued for by applying it to well-known models and experimental results. In Chapter 5 it is shown how our interpretation solves the quantum measurement problem, both in the ideal and in the non-ideal versions, and in Chapter 6 the compatibility between this solution and the environment-induced decoherence approach is proved. Finally, in Chapter 7 the quantum ontology implied by the interpretation is described from a philosophical viewpoint.

Chapter 2

INTERPRETATIVE POSTULATES

During the last decades, the research on the mathematical properties of the formal structure of quantum mechanics has shown a great advance: many results, unknown by the founding fathers of the theory, have been obtained, and this work has greatly improved the understanding of the deep obstacles that any interpretation must face. However, this interest in the features of the formalism has led to forget the physical content of the theory: in the last times, usually the arguments rely on mathematical results and discussions center around the formal models of the quantum measurement. But quantum mechanics is a *physical* theory that has been applied to many well-known systems and by means of which an impressive amount of experimental evidence has been accounted for. Therefore, a "good" interpretation of quantum mechanics should not only face the traditional interpretational challenges of the theory, but also show its agreement with the orthodox practice of physics. In this sense, our proposal moves away from the present trend in the subject by placing an element with a clear physical meaning, the Hamiltonian of the system, at the heart of the interpretation.

2.1. SYSTEMS AND SUBSYSTEMS

In order to study the physical world, we have to identify the systems populating it. We can cut out the physical reality in many different and arbitrary ways, but only when a portion of reality does not interact with others, we have a non-arbitrary, objective criterion to identify that portion as a system.

For this reason, we design our interpretation to account for those pieces of reality non-interacting with other pieces and, so, we conceive only closed systems as quantum systems. On this basis, and by adopting an algebraic perspective, we define

> **Systems Postulate (SP):** A quantum system $\mathcal{S}$ is represented by a pair $(\mathcal{O}, H)$ such that (i) $\mathcal{O}$ is a space of self-adjoint operators on a Hilbert space $\mathcal{H}$, representing the observables of the system, (ii) $H \in \mathcal{O}$ is the time-independent Hamiltonian of the system $\mathcal{S}$, and (iii) if $\rho_0 \in \mathcal{O}'$ (where $\mathcal{O}'$ is the dual space of $\mathcal{O}$) is the initial state of $\mathcal{S}$, it evolves according to the Schrödinger equation in its von Neumann version.

Of course, any quantum system can be decomposed in parts in many ways; however, not any decomposition will lead to parts which are, in turn, quantum systems. This will be the case only when the components' behaviors are dynamically independent to each other, that is, the time-evolution of each component can be described independently to the remaining ones (see Harshman and Wickramasekara 2007). In other words, a quantum system can be split into subsystems when there is no interaction among the subsystems. Then, we can say that

> **Decomposition Postulate (DP):** A quantum system represented by $\mathcal{S}\colon (\mathcal{O}, H)$, with initial state $\rho_0 \in \mathcal{O}'$, is *composite* when it can be partitioned into two quantum systems represented by $\mathcal{S}^1\colon (\mathcal{O}^1, H^1)$ and $\mathcal{S}^2\colon (\mathcal{O}^2, H^2)$ such that (i) $\mathcal{O} = \mathcal{O}^1 \otimes \mathcal{O}^2$, and (ii) $H = H^1 \otimes I^2 + I^1 \otimes H^2$, (where I^1 and I^2 are the identity operators in the corresponding tensor product spaces). In this case, the initial states of $\mathcal{S}^1$ and $\mathcal{S}^2$ are obtained as the partial traces $\rho_0^1 = Tr_2 \rho_0$ and $\rho_0^2 = Tr_1 \rho_0$; we say that $\mathcal{S}^1$ and $\mathcal{S}^2$ are *subsystems* of the composite system, $\mathcal{S} = \mathcal{S}^1 \cup \mathcal{S}^2$. If the system is not composite, it is *elemental*.

This definition of composite system does not imply that the initial state ρ_0 of $\mathcal{S}$ is the tensor product $\rho_0^1 \otimes \rho_0^2$: this factored or uncorrelated state is a very special kind of state corresponding to independent preparations of the component systems (see Ballentine 1998). On the contrary, in the general case the initial state is a correlated or entangled state $\rho_0 \in \mathcal{O}'$; nevertheless, since there is no interaction between $\mathcal{S}^1$ and $\mathcal{S}^2$, $\left[H^1 \otimes I^2, I^1 \otimes H^2\right] = 0$ and, then,

$$exp\left[-iHt/\hbar\right] = exp\left[-iH^1t/\hbar\right] exp\left[-iH^2t/\hbar\right] \tag{2-1}$$

Therefore,

$$\begin{aligned} \rho^1(t) &= Tr_2\rho(t) = Tr_2\left[e^{-iHt/\hbar}\rho_0 e^{iHt/\hbar}\right] = \\ &= e^{iH^1t/\hbar}\left[Tr_2\rho_0\right]e^{-iH^1t/\hbar} = e^{iH^1t/\hbar}\rho_0^1 e^{-iH^1t/\hbar} \end{aligned} \tag{2-2}$$

$$\begin{aligned} \rho^2(t) &= Tr_1\rho(t) = Tr_1\left[e^{-iHt/\hbar}\rho_0 e^{iHt/\hbar}\right] = \\ &= e^{iH^2t/\hbar}\left[Tr_1\rho_0\right]e^{-iH^2t/\hbar} = e^{iH^2t/\hbar}\rho_0^2 e^{-iH^2t/\hbar} \end{aligned} \tag{2-3}$$

This means that, in spite of the correlations, the subsystems $\mathcal{S}^1$ and $\mathcal{S}^2$ are *dynamically independent*: each one of them will evolve under the action of its own Hamiltonian.

It is quite clear that the decomposition of a quantum system into subsystems is not always possible: it may happen that there is no partition of the whole $\mathcal{S}$ such that the total Hamiltonian can be expressed as a sum of component Hamiltonians. On the contrary, a composite system can always be defined on the basis of the component systems:

> **Composition Postulate (CP):** Given two quantum systems represented by $\mathcal{S}^1$: $(\mathcal{O}^1, H^1)$ and $\mathcal{S}^2$: $(\mathcal{O}^2, H^2)$, with initial states $\rho_0^1 \in \mathcal{O}^{1\prime}$ and $\rho_0^2 \in \mathcal{O}^{2\prime}$ respectively, a quantum system represented by $\mathcal{S}$: $(\mathcal{O}, H)$ with initial state $\rho_0 \in \mathcal{O}'$ can always be defined, such that:
> (i) $\mathcal{O} = \mathcal{O}^1 \otimes \mathcal{O}^2$,
> (ii) $H = H^1 \otimes I^2 + I^1 \otimes H^2 + H_{\text{int}} \in \mathcal{O}$, where H_{int} is called *interaction Hamiltonian*, and

(iii) $\rho_0 = \rho_0^1 \otimes \rho_0^2 \in \mathcal{O}'$.

In this case, the initial state ρ_0 of $\mathcal{S}$ and the initial states ρ_0^1 of $\mathcal{S}^1$ and ρ_0^2 of $\mathcal{S}^2$ are still related by a partial trace, since

$$\rho_0^1 = Tr_2\rho_0 = Tr_2\left(\rho_0^1 \otimes \rho_0^2\right) \quad \rho_0^2 = Tr_1\rho_0 = Tr_1\left(\rho_0^1 \otimes \rho_0^2\right) \quad (2\text{-}4)$$

However, when the two systems $\mathcal{S}^1$ and $\mathcal{S}^2$ interact to each other, then $H_{\text{int}} \neq 0$ and, therefore, ρ_0^1 and ρ_0^2 do not evolve unitarily according to the Schrödinger equation. This means that, strictly speaking, $\mathcal{S}^1$ and $\mathcal{S}^2$ are not subsystems of $\mathcal{S}$ but should be considered as mere "parts" of $\mathcal{S}$; we describe this situation as $\mathcal{S}= \mathcal{S}^1 + \mathcal{S}^2$. Only in the particular case that $H_{\text{int}} = 0$, $\mathcal{S}^1$ and $\mathcal{S}^2$ will evolve unitarily as in eqs. (2-2) and (2-3), and they will properly be subsystems of $\mathcal{S}= \mathcal{S}^1 \cup \mathcal{S}^2$.

2.2. ACTUALIZATION RULE

As it is well known, the Kochen-Specker theorem (Kochen and Specker 1967) is an insurmountable barrier to any classical-like interpretation of quantum mechanics: it shows the contextuality of the theory by proving the impossibility of ascribing actual values to all the observables of a quantum system in a non-contradictory manner. Therefore, any realist, non-collapse interpretation is committed to select a preferred context, which defines the observables that will acquire actual values. This point is so relevant that Bub (1997) classifies the best known realist non-collapse interpretations in terms of the observable R chosen to define the preferred context in each one of them. For instance, in Bohr's complementarity interpretation (Bohr 1948), R is defined by the experimental arrangement; in the interpretation of Bohm (1952), R is simply the position observable; for some modal interpretations (Kochen 1985, Dieks 1988, Vermaas and Dieks 1995), R is given by the biorthonormal decomposition (Schmidt) theorem. When the many interpretations of quantum mechanics proposed through the years are reviewed from this general viewpoint, it is easy to realize that the Hamiltonian has been systematically ignored in the discussions. In our interpretation, on the

contrary, the Hamiltonian of the system will be decisive in the definition of the preferred context.

Let us recall that quantum mechanics is a probabilistic theory: by contrast with classical mechanics, it does not ascribe actual values to the observables of the system, but only probabilities to each possible value (see Dieks 2007). Moreover, quantum mechanics is intrinsically probabilistic: any attempt to ascribe actual values to all the observables of the system, in such a way that probabilities be interpreted in terms of ignorance about an underlying classical-like state, runs into contradiction as a consequence of the Kochen-Specker theorem (see Hughes 1989). This means that the rule of actual-value ascription, which selects the preferred context, cannot be inferred from the formalism, but has to be introduced as an interpretative postulate. Therefore, the adequacy of such a rule has to be assessed in the light of its physical relevance and its ability to solve the interpretational problems of the theory.

Since we have defined a quantum system as a closed system, its energy is constant in time and, then, the Hamiltonian H is time-independent: H is always invariant under time-displacement. Nevertheless, in a given quantum system H may have other symmetries or not. To say that the Hamiltonian is symmetric or invariant under a certain continuous transformation means that

$$e^{iKs} H\, e^{-iKs} = H \text{ , then } \left[H, K\right] = 0 \qquad (2\text{-}5)$$

where s is the parameter of the transformation and K is the corresponding generator. This means that, when H is invariant under a certain transformation, the generator of that transformation is a constant of motion: each symmetry of H defines a conserved quantity. For instance, the invariance of H under space-displacement in the direction α implies that the component P_α of the momentum P is a constant of motion; the invariance of H under space-rotation about an axis α implies that the component J_α of the total angular momentum J is a constant of motion.

Moreover, we know that each symmetry of the Hamiltonian leads to an energy degeneracy. In fact, if H is invariant under a symmetry transformation with generator K, we can write,

$$K H \left|n\right\rangle = K\omega_n \left|n\right\rangle \qquad \Rightarrow H K \left|n\right\rangle = \omega_n K \left|n\right\rangle \qquad (2\text{-}6)$$

This means that any vector $K|n\rangle$ obtained by applying the operator K to the eigenvector $|n\rangle$ is also an eigenvector of H with the same eigenvalue ω_n. If H is expressed as $H = \sum_n \omega_n P_n$, where P_n is the eigenprojector corresponding to the eigenvalue ω_n, we can write explicitly the index k_n corresponding to the degeneracy of ω_n in such a way that

$$H|n,k_n\rangle = \omega_n |n,k_n\rangle \Rightarrow H = \sum_n \omega_n \sum_{k_n} |n,k_n\rangle\langle n,k_n| \tag{2-7}$$

$$K|n,k_n\rangle = \kappa_{k_n} |n,k_n\rangle \Rightarrow K = \sum_n \sum_{k_n} \kappa_{k_n} |n,k_n\rangle\langle n,k_n| \tag{2-8}$$

The degeneracies with origin in symmetries are called “normal” (Tinkham 1964) or “systematic” (Cohen-Tannoudji, Diu and Laloë 1977). On the contrary, degeneracies that have no obvious origin in symmetries are called “accidental”. However, deeper study usually shows either that the accidental degeneracy is not exact, or else that a hidden symmetry in the Hamiltonian can be found which explains the degeneracy. For this reason it is assumed that, once all the symmetries of the Hamiltonian have been considered, a basis for the Hilbert space of the system is obtained and the “good quantum numbers” are well defined.

Now we have all the conceptual elements necessary to present our rule of actual-value ascription. We shall call it ‘Actualization Rule’ because it is the interpretative postulate that defines, among all the observables of the system, which of them acquire actual, and not merely possible, values. The basic idea can be expressed by the classical Latin maxim “*Ubi lex non distinguit, nec nos distinguere debemus*”: where the law does not distinguish, neither ought we to distinguish. The Hamiltonian of the system, with its symmetries, is what rules actualization; then, none observable whose eigenvalues would distinguish among eigenvectors of a single degenerate eigenvalue of the Hamiltonian has to acquire an actual value, since that value would introduce in the system an asymmetry not contained in the Hamiltonian. Once this basic idea has been clearly understood, the Actualization Rule can be easily formulated.

> **Actualization Rule (AR):** Given an elemental quantum system represented by $\mathcal{S}: (\mathcal{O}, H)$, if $H = 0$, there is no actualization, but if $H \neq 0$, the actual-valued observables of

$\mathcal{S}$ are H and all the observables commuting with H and having, at least, the same symmetries as H.

Let us see how the rule works in different cases:

(a) The Hamiltonian H does not have symmetries; this means that it is non-degenerate. In this case,

$$H|n\rangle = \omega_n |n\rangle \text{ with } \omega_n \neq \omega_{n'} \tag{2-9}$$

where $\{|n\rangle\}$ is a basis of the Hilbert space $\mathcal{H}$. Therefore, n is the only good quantum number: the actual-valued observables of the system are H and all the observables commuting with H.

(b) The Hamiltonian H has certain symmetries that lead to energy degeneracy. In this case, H can be written as

$$H|n,i_n\rangle = \omega_n |n,i_n\rangle \Rightarrow H = \sum_n \omega_n \sum_{i_n} |n,i_n\rangle\langle n,i_n| = \sum_n \omega_n P_n \tag{2-10}$$

where $\omega_n \neq \omega_{n'}$ and the index i_n expresses the degeneracy of the energy eigenvalue ω_n. Let us consider an observable of the form

$$A = \sum_{n,i_n} a_n |n,i_n\rangle\langle n,i_n| = \sum_n a_n \sum_{i_n} |n,i_n\rangle\langle n,i_n| = \sum_n a_n P_n \tag{2-11}$$

where $a_n \neq a_{n'}$ It is clear that $[H,A]=0$. Moreover, A has the same degeneracy as H since they have the same eigenprojectors P_n: the subspace spanned by the degenerate eigenvectors corresponding to a_n is the same as that spanned by the degenerate eigenvectors corresponding to ω_n. In other words, A has the same symmetries as H. Therefore, all the observables A commuting with H and having the form of eq. (2-11) are actual-valued. On the contrary, e.g., observables of the form

$$B = \sum_{n,i_n} b_{n,i_n} |n,i_n\rangle\langle n,i_n| \tag{2-12}$$

in spite of commuting with H, do not acquire actual values, since the actualization of a particular eigenvalue of B would discriminate among the degenerate eigenvectors corresponding to a single degenerate eigenvalue ω_n of H and, in this way, it would introduce in the system an asymmetry not contained in the Hamiltonian.

(c) An interesting particular case arises when all the eigenvalues ω_n have the same i-fold degeneracy: the index i, that expresses the energy degeneracy, is not a function of n. Then, in this case eq. (2-10) becomes

$$H|n,i\rangle = \omega_n |n,i\rangle \tag{2-13}$$

As a consequence, the Hamiltonian can be decomposed as

$$H = \sum_n \omega_n \sum_i |n,i\rangle\langle n,i| = \sum_n \omega_n |n\rangle\langle n| \otimes \sum_i |i\rangle\langle i| = H^{ND} \otimes I^D \tag{2-14}$$

This equation expresses the decomposition of the original system $\mathcal{S}$ into two non-interacting subsystems $\mathcal{S}^{ND}$ and $\mathcal{S}^D$:

- The system $\mathcal{S}^{ND}$ is represented in the Hilbert space $\mathcal{H}^{ND}$, with basis $\{|n\rangle\}$, and its Hamiltonian H^{ND} is non-degenerate.
- The system $\mathcal{S}^D$ is represented in the Hilbert space $\mathcal{H}^D$, with basis $\{|i\rangle\}$, and its Hamiltonian is $H^D = 0$.

Therefore, the original system $\mathcal{S}$ is a composite system $\mathcal{S} = \mathcal{S}^{ND} \cup \mathcal{S}^D$ such that $\mathcal{H} = \mathcal{H}^{ND} \otimes \mathcal{H}^D$ and

$$H = H^{ND} \otimes I^D + I^{ND} \otimes H^D + H_{\text{int}} = H^{ND} \otimes I^D \tag{2-15}$$

where $H^D = H_{\text{int}} = 0$. As a consequence, the Actualization Rule has to be applied to each elemental subsystem:

- In $\mathcal{S}^{ND}$ the preferred basis is $\{|n\rangle\}$: the actual-valued observables are H^{ND} and all the observables belonging to $\mathcal{H}^{ND} \otimes \mathcal{H}^{ND}$ and commuting with H^{ND}.

- In $\mathcal{S}^D$ there is no actualization because $H^D = 0$: the observables of $\mathcal{S}^D$ do not acquire actual values.

As we have said, Bub (1997) classifies the realist non-collapse interpretations in terms of the observable R by means of which the preferred context is defined. In particular, each interpretation selects a sublattice $\mathcal{D}(|e\rangle, R)$ of the complete lattice of quantum propositions, where $|e\rangle$ is the instantaneous state of the system: in this sublattice, truth values can be assigned and standard Kolmogorov probabilities can be defined. But since, in general, the sublattice depends on the instantaneous state $|e\rangle$, it changes with time. This means that the set of actual-valued observables is different at each time as the instantaneous state of the system evolves dynamically. This result not only defies intuitions (a system having, say, position but not momentum at t, and momentum but not position at an infinitesimal time later), but also leads to the need of accounting for the dynamics of actual properties (Dieks and Vermaas 1998, Bacciagaluppi and Dickson 1999). In our interpretation, on the contrary, this step is unnecessary because the dynamics of actual properties is trivial. In fact, since in any case the actual-valued observables commute with the Hamiltonian, they are constants of motion of the system: in spite of the fact that probabilities are continuously evolving, the set of actual-valued observables is time-independent and, thus, completely robust. This supplies a more intuitive picture of the behavior of a quantum system: actualization occurs only once, with the constitution of the system as such, and since then the actual-valued properties are the same at any time, up to the time when the system "disappears" as that particular system by interacting with another system. This picture is consistent with the well-known fact that, if the energy of a quantum system is completely definite, time is completely indefinite; therefore, the search of a definite time when actualization occurs in a closed, constant-energy system is senseless.

Chapter 3

INTERPRETATION AND GALILEAN GROUP

Although it is usual to read that non-relativistic quantum mechanics is invariant under the Galilean transformations, this issue has been scarcely treated in the standard literature on the theory (see the complaint by Lévi-Leblond 1974). This fact has its counterpart in the field of the interpretation of quantum mechanics: the relevance of the Galilean group is rarely discussed in the impressive amount of literature on the subject. However, the relationship between interpretation and Galilean transformations deserves to be seriously analyzed: the fact that the theory is invariant under the Galilean group does not guarantee the same property for the interpretation since, in general, interpretations add interpretative postulates to the formal structure of the theory.

3.1. THE GALILEAN GROUP

The space-time symmetry group of non-relativistic –classical or quantum– mechanics is the Galilean group, defined by ten symmetry generators K_α, with $\alpha = 1 \text{ to } 10$: one time displacement K_τ, three space-displacements K_{ρ_i}, three space-rotations K_{θ_i}, and three boost-velocity components K_{u_i}, with $i = x, y, z$. The Galilean group is a Lie group with its associated Galilean algebra of generators. The central extension of the Galilean algebra is obtained as a semi-direct product between the Galilean algebra and the algebra

generated by a central charge, which in this case denotes the mass operator $M = mI$, where I is the identity operator and m is the mass (see Weinberg,1995, Bose 1995). In this central extension, when there are not external fields acting on the system, the symmetry generators represent the basic magnitudes of the theory: the energy $H = \hbar K_\tau$, the three momentum components $P_i = \hbar K_{\rho_i}$, the three angular momentum components $J_i = \hbar K_{\theta_i}$, and the three boost components $G_i = \hbar K_{u_i}$. The rest of the physical magnitudes can be defined in terms of these basic ones: for instance, the three position components are $Q_i = G_i / m$, the three orbital angular momentum components are $L_i = \varepsilon_{ijk} Q_j P_k$ (where ε_{ijk} is the Levi-Civita tensor, such that $i \neq k$, $j \neq k$, $\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij} = 1$, $\varepsilon_{ikj} = \varepsilon_{jik} = \varepsilon_{kji} = -1$), and the three spin components are $S_i = J_i - L_i$. In order to simplify the presentation, from now on we shall use the expression 'Galilean group' and 'Galilean algebra' to refer to the corresponding central extension, and we shall take $\hbar = 1$.

The Galilean group is defined by the commutation relations between its generators:

(a) $\left[P_i, P_j\right] = 0$ (f) $\left[G_i, P_j\right] = i\delta_{ij} M$

(b) $\left[G_i, G_j\right] = 0$ (g) $\left[P_i, H\right] = 0$

(c) $\left[J_i, J_j\right] = i\varepsilon_{ijk} J_k$ (h) $\left[J_i, H\right] = 0$

(d) $\left[J_i, P_j\right] = i\varepsilon_{ijk} P_k$ (i) $\left[G_i, H\right] = iP_i$

(e) $\left[J_i, G_j\right] = i\varepsilon_{ijk} G_k$ (3-1)

Moreover, each Galilean transformation T_α acts on observables and states as

$$O \to O' = U_{s_\alpha} O U_{s_\alpha}^{-1} \quad \left|\varphi\right\rangle \to \left|\varphi'\right\rangle = U_{s_\alpha} \left|\varphi\right\rangle \tag{3-2}$$

where s_α is the parameter corresponding to the transformation T_α, and U_{s_α} is the family of unitary operators describing T_α. Since in any case s_α is a continuous parameter, each U_{s_α} can be expressed in terms of the corresponding symmetry generator K_α as

$$U_{s_\alpha} = e^{iK_\alpha s_\alpha} \tag{3-3}$$

The combined action of all the transformations is given by

$$U_{s_\alpha} = \prod_{\alpha=1}^{10} e^{iK_\alpha s_\alpha} \tag{3-4}$$

When the state vector is represented as a function of space-time coordinates, there is an inverse relation between transformations on function space and transformations on coordinates (see Ballentine 1998):

$$\varphi(\boldsymbol{x},t) = U_s\, \varphi(\boldsymbol{x}',t') \tag{3-5}$$

In the case of time-displacement, the transformation is the transformation $(\boldsymbol{x},t_0) \to (\boldsymbol{x},t_0+\tau)$ is $t_0 \to t_0+\tau$ and U_s is $U_\tau = e^{iH\tau}$, where the Hamiltonian H is the generator of the transformation and τ is the corresponding continuous parameter:

$$\left|\varphi(t_0)\right\rangle = e^{iH\tau}\left|\varphi(t_0+\tau)\right\rangle \tag{3-6}$$

Then, by making $t_0 = 0$ and $\tau = t$, we obtain

$$\left|\varphi(t)\right\rangle = e^{-iHt}\left|\varphi(0)\right\rangle \tag{3-7}$$

This equation, which has the form of a solution of the Schrödinger equation, can be obtained only when H is independent of t and, as a consequence, it is the generator of time-displacements. This means that the Schrödinger equation has the physical meaning of describing time-displacements only for time-independent Hamiltonians, that is, for closed systems. On the other hand, H may have the remaining space-time symmetries or not. As we have seen, to say that the Hamiltonian is symmetric or invariant under a certain continuous transformation T_α means that $[H,K_\alpha] = 0$ and, therefore, K_α is a constant of motion of the system (see eq. (2-5)).

3.2. THE INVARIANCE OF THE SCHRÖDINGER EQUATION

As we have seen, when there are no external fields acting on the system, the Galilean group is defined by the commutation relations (3-1). Now we can consider how the Galilean transformations affect the Schrödinger equation,

$$\frac{d\left|\varphi\right\rangle}{dt} = -iH\left|\varphi\right\rangle \tag{3-8}$$

Let us premultiply both members of the equation by $U = e^{iKs}$; by using the property $UU^{-1} = I$ and, then, adding $\left(dU/dt\right)\left|\varphi\right\rangle$ to both members, we obtain

$$U\frac{d\left(\left|\varphi\right\rangle\right)}{dt} + \frac{dU}{dt}\left|\varphi\right\rangle = -iUHU^{-1}U\frac{d\left|\varphi\right\rangle}{dt} + \frac{dU}{dt}\left|\varphi\right\rangle \tag{3-9}$$

Therefore,

$$\frac{d\left(U\left|\varphi\right\rangle\right)}{dt} = -i\left[UHU^{-1} + i\frac{dU}{dt}U^{-1}\right]U\left|\varphi\right\rangle \tag{3-10}$$

If we recall the action of the Galilean transformations on states and observables (see eqs. (3-2), we can write

$$\frac{d\left|\varphi'\right\rangle}{dt} = -i\left[H' + i\frac{dU}{dt}U^{-1}\right]\left|\varphi'\right\rangle \tag{3-11}$$

In a closed, constant-energy system free from external fields, H is time-independent and the P_i and the J_i are constants of motion (see eqs. (3-1g,h)). Then, for time-displacements, space-displacements and space-rotations, $dU/dt = de^{iKs}/dt = 0$, where K and s stand for H and τ, P_i and ρ_i, and J_i and θ_i, respectively. As a consequence, eq. (3-11) yields

$$\frac{d\left|\varphi'\right\rangle}{dt} = -iH'\left|\varphi'\right\rangle \tag{3-12}$$

Moreover, for those transformations, $H' = H$ because:

- Time-displacements: $H' = e^{iH\tau}He^{-iH\tau} = H$ since $\left[H,H\right] = 0$
- Space-displacements: $H' = e^{iP_i\rho_i}He^{-iP_i\rho_i} = H$ since $\left[P_i,H\right] = 0$ (relation (3-1g))
- Space-rotations: $H' = e^{iJ_i\theta_i}He^{-iJ_i\theta_i} = H$ since $\left[J_i,H\right] = 0$ (relation (3-1h))

By applying these results to eq. (3-12), we prove the invariance of the Schrödinger equation under time-displacements, space-displacements and space-rotations when there are no external fields acting on the system:

$$\frac{d\left|\varphi'\right\rangle}{dt} = -iH\left|\varphi'\right\rangle \tag{3-13}$$

The case of boost-transformations is different from the previous cases, because the Hamiltonian is not boost-invariant even when the system is free from external fields (for the same claim in classical mechanics, see Butterfield 2007). In fact, under a boost-transformation corresponding to a velocity u_x, H changes as (see relation (3-1i))

$$H' = e^{iG_xu_x}He^{-iG_xu_x} \neq H \text{ since } \left[G_x,H\right] = iP_x \neq 0 \tag{3-14}$$

and the generator G_x is

$$G_x = mQ_x = m\left(Q_{x0} + V_xt\right) = mQ_{x0} + P_xt \tag{3-15}$$

Since G_x is not time-independent, $dU/dt = de^{iG_xu_x}/dt \neq 0$, and eq. (3-11) yields

$$\frac{d\left|\varphi'\right\rangle}{dt} = -i\left(H' + i\frac{de^{iG_xu_x}}{dt}e^{-iG_xu_x}\right)\left|\varphi'\right\rangle \tag{3-16}$$

In order to know the value of the bracket in the r.h.s. side of eq. (3-16), we have to compute both terms in the bracket. By using the Hadamard lemma applied to the Baker-Campbell-Hausdorff formula, $e^{B}Ae^{-B} = A + \left[B, A\right] + \\ + (1/2!)\left[B,\left[B, A\right]\right] + (1/3!)\left[B,\left[B,\left[B, A\right]\right]\right] + \cdots$, and by applying the commutation relations (3-1i) and (3-1f), H' results

$$H' = e^{iG_x u_x} H e^{-iG_x u_x} = H - u_x P_x + \frac{1}{2} M u_x^2 = H + T_B \tag{3-17}$$

where T_B is the boost contribution to the energy. In turn, by means of the lemma and the commutation relation (3-1f), P_x' results

$$P_x' = e^{iG_x u_x} P_x e^{-iG_x u_x} = P_x - M u_x \;\Rightarrow\; P' = P + P_B \tag{3-18}$$

where $P_B = \left(-M u_x, 0, 0\right)$ is the boost contribution to the momentum. In turn, when there are no external fields, the Hamiltonian can be written in terms of the internal energy W as

$$H = \frac{P^2}{2m} + W \tag{3-19}$$

By means of eqs. (3-18) and (3-19), it is easy to show that the transformed Hamiltonian can be expressed as

$$H' = \frac{\left(P + P_B\right)^2}{2m} + W \tag{3-20}$$

On the other hand, we have to compute the time-derivative $de^{iG_x u_x}/dt$ of eq. (3-16). By using the identity $e^{A+B} = e^{A} e^{B} e^{-[A,B]/2}$ which holds when $\left[A,\left[A, B\right]\right] = \left[B,\left[A, B\right]\right] = 0$, and by applying the commutation relation $\left[P_i, F(Q_j)\right] = -i\,\partial F / \partial Q_j$ valid on the Galilean algebra, it can be proved that

$$\frac{de^{iG_x u_x}}{dt} = -i\left(u_x P_x - \frac{1}{2} M u_x^2\right) e^{iG_x u_x} \tag{3-21}$$

When the results (3-17) and (3-21) are introduced into eq. (3-16), the terms added to H in H' cancel with those coming from the term containing the time-derivative; so, we prove the invariance of the Schrödinger equation also for boost-transformations:

$$\frac{d\left|\varphi'\right\rangle}{dt} = -iH\left|\varphi'\right\rangle \tag{3-22}$$

Summing up, when there are no external fields acting on the system, the Hamiltonian is invariant under time-displacements, space-displacements and space-rotations, but not under boost-transformations. In spite of this fact, the Schrödinger equation is completely invariant under the Galilean group, and this conceptually means that the state vector $\left|\varphi\right\rangle$ does not "see" the effect of the transformations: the evolutions of $\left|\varphi\right\rangle$ and $\left|\varphi'\right\rangle$ are identical. In other words, the time-behavior of the system is independent of the reference frame used for the description.

3.3. THE INVARIANCE OF THE MODAL-HAMILTONIAN INTERPRETATION

Let us recall the physical meaning of a symmetry transformation. A continuous transformation, as in the case of the Galilean group, admits two interpretations. Under the active interpretation, the transformation corresponds to a change from one system to another –transformed– system; under the passive interpretation, the transformation consists in a change of the viewpoint –reference frame– from which the system is described (see Brading and Castellani 2007). Nevertheless, in both cases the validity of a group of symmetry transformations expresses the fact that the identity and the behavior of the system are not altered by the application of the transformations: in the active interpretation language, the original and the transformed systems are equivalent; in the passive interpretation language, the original and the transformed reference frames are equivalent.

In the case of the Galilean group, and adopting the passive interpretation language, the validity of the group amounts to the equivalence between reference frames time-displaced, space-displaced or space-rotated with respect

to each other, and between inertial reference frames: the application of a Galilean transformation does not introduce a modification in the physical situation, but only expresses a change of the perspective from which the system is described. It is quite clear that any adequate interpretation of quantum mechanics should not violate this physical meaning of the Galilean transformations. In particular, if a realist interpretation determines the preferred context that selects the actual-valued observables of the system, such a context should not change under the transformations of the group: from a realist viewpoint, it would be unacceptable that the set of actual-valued observables were different as the mere result of a change in the reference frame from which the system is described (Brown, Suárez and Bacciagaluppi 1998). Therefore, one is entitled to ask whether the MHI satisfies this constraint.

As we have seen, the preferred context selected by the modal-Hamiltonian Actualization Rule only depends on the Hamiltonian of the system. Then, at first sight, the requirement of invariance of the preferred context under the Galilean transformations would amount to the requirement of invariance for the Hamiltonian. It is easy to see that this requirement is fulfilled in the case of time-displacement, space-displacement and space-rotation, since the Hamiltonian is invariant under those transformations. However, it is not clear that the requirement of invariance of the preferred context completely holds, since the Hamiltonian is not invariant under boost-transformations. This seems to lead to the conclusion that the preferred context selected when the system is described in the reference frame RF is different that the preferred context selected in the boost-transformed reference frame RF': the set of actual-valued observables would change in different inertial frames, and this fact would make an objective feature of the system to depend on the particular descriptive viewpoint adopted. Of course, this conclusion would be unacceptable on physical grounds. Nevertheless, the seeming conflict can be solved when the MHI is considered as a whole.

For simplicity, let us consider a system $\mathcal{S}$ described in the reference frame RF_0 at rest with respect of the center of mass of $\mathcal{S}$, in such a way that its Hamiltonian is H_0. In RF_0 the momentum P_0 of the system is zero and, as a consequence,

$$P_0 = 0 \quad \Rightarrow \quad H_0 = \frac{P_0^2}{2m} + W = W \tag{3-23}$$

Let us now consider the system $\mathcal{S}$ described in a reference frame RF_1 in motion with a constant velocity u_x with respect to RF_0. T he new Hamiltonian H_1 is (see eq. (3-17))

$$H_1 = H_0 + T_B = W + \frac{1}{2} M u_x^2 \tag{3-24}$$

This means that, in RF_1, H_1 is the sum of two terms: a Hamiltonian $H_0 = W$ relative to the center of mass of the system, and a Hamiltonian $H_K = T_B$ representing the total kinetic energy of translation. In turn, since $H_0 = W$ does not depend on the position nor on the momentum of the center of mass, but only on the differences of positions and their respective conjugate momenta, and $H_K = T_B$ only depends on the boost-velocity, we can guarantee that $\left[H_0, H_K\right] = 0$. If $\mathcal{H}^Q$ is the Hilbert space of the wavefunctions of the differences of coordinates and $\mathcal{H}^P$ is the Hilbert space of the wavefunctions of the coordinates of the center of mass, then H_1 can be expressed as

$$H_1 = H_0 + H_K = H_W^Q \otimes I^P + I^Q \otimes H_K^P \tag{3-25}$$

where H_W^Q is the internal energy Hamiltonian acting on the Hilbert space $\mathcal{H}^Q$, H_K^P is the kinetic Hamiltonian acting on the Hilbert space $\mathcal{H}^P$, and I^Q and I^P are the identity operators on the respective spaces. But, according to the modal-Hamiltonian definition of elemental and composite system DP, eq. (3-25) expresses the fact that $\mathcal{S}$ is a composite system, whose elemental subsystems are:

- a system $\mathcal{S}_0$, defined by the Hamiltonian H_W^Q relative to the center of mass, which represents the internal energy W.
- a system $\mathcal{S}_K$, defined by the Hamiltonian H_K^P representing the translational kinetic energy.

In turn, the modal-Hamiltonian Actualization Rule applies to elemental quantum systems. This means that, according to the MHI, both elemental systems, $\mathcal{S}_0$ and $\mathcal{S}_K$, "actualize" independently:

- In $\mathcal{S}_0$, the Hamiltonian H_W^Q (or $H_0 = W$) determines the set of actual-valued observables, which is the same set selected by the Actualization Rule in the reference frame RF_0.
- In $\mathcal{S}_K$, the Hamiltonian H_K^P (or $H_K = T_B$) acquires an actual value and, with it, the total kinetic energy of translation T_B also turns out to be actual-valued.

This shows that, by contrast to what originally supposed, in the context of the MHI a boost-transformation does not modify the preferred context in a way that violates the physical meaning of the transformation. The only change resulting from passing from an inertial frame to another consists in the appearance of an elemental system $\mathcal{S}_K$, non-interacting with $\mathcal{S}_0$, where the kinetic energy acquires an actual value (we shall come back to this point in Chapter 7, Section 7.4).

3.4. ACTUALIZATION RULE AND CASIMIR OPERATORS

We have shown that, when the system is free from external fields, a boost-transformation only introduces a change in the subsystem that carries the kinetic energy of translation: the internal energy remains unaltered under the transformation. This should not sound surprising to the extent that the internal energy is a Casimir operator of the –central extension of the– Galilean group.

A Casimir operator of a Lie group is an operator that commutes with all the generators of the group and, therefore, is invariant under all the transformations of the group (see Tung 1985). The Galilean group has three Casimir operators: the mass operator M, the operator S^2, and the internal energy operator $W = H - P^2/2m$. The eigenvalues of the Casimir operators label the irreducible representations of the group (see Wigner 1939, Bargman 1954, Lévi-Leblond 1963). So, in each irreducible representation, the Casimir operators are multiples of the identity: $M = mI$, where m is the mass, $S^2 = s(s+1)I$, where s is the eigenvalue of the spin S, and $W = wI$, where w is the scalar internal energy

We have also pointed out that, under the passive interpretation, the application of a Galilean transformation expresses a change in the perspective from which the system is described. Then, any realist interpretation should agree with this physical fact: the rule of actual-value ascription should select a set of actual-valued observables that remains unaltered under the transformations. Since the Casimir operators of the Galilean group are invariant under all the transformations of the group, one can reasonably expect that those Casimir operators belong to the set of the actual-valued observables. Then, the Actualization Rule can be reformulated as follows:

> **Actualization Rule' (AR'):** Given a quantum system free from external fields and represented by $\mathcal{S}: (\mathcal{O}, H)$, its actual-valued observables are the observables C_i represented by the Casimir operators of the Galilean group in the corresponding irreducible representation, and all the observables commuting with the C_i and having, at least, the same symmetries as the C_i.

Since the Casimir operators of the Galilean group are M, S^2 and W, this reformulation of the rule is in agreement with the original AR when applied to a system free from external fields:

- The actual-valuedness of M and S^2 postulated by AR' follows from AR: these observables commute with H and do not break its symmetries because, in non-relativistic quantum mechanics, both are multiples of the identity in any irreducible representation. The fact that M and S^2 always acquire actual values is completely natural from a physical viewpoint, since mass and spin are properties supposed to be always possessed by any quantum system and measurable in any physical situation.
- The actual-valuedness of W might seem to be in conflict with AR because W is not the Hamiltonian: whereas W is Galilean-invariant, H changes under the action of a boost. However, as we have seen, this is not a real obstacle when the elemental subsystems to which AR applies are considered.

In addition to supplying an explicitly invariant version of the rule of actual-value assignment, AR' leads us to a final reflection. The identity and the behavior of any quantum system free from external fields must remain unchanged under the action of the Galilean group. On the other hand, from a

realist viewpoint, the fact that certain observables acquire an actual value is an objective fact in the behavior of the system; therefore, the set of actual-valued observables selected by a realist interpretation must be also Galilean-invariant. But the Galilean-invariant observables are always functions of the Casimir operators of the Galilean group. As a consequence, one is led to the conclusion that any realist interpretation that intends to preserve the objectivity of actualization may not stand very far from our MHI.

Summing up, the modal-Hamiltonian Actualization Rule mirrors the Galilean-invariance of the Schrödinger equation: when the Schrödinger equation is invariant –no external fields acting on the system–, the rule is also invariant when expressed in terms of the Casimir operators of the Galilean group. This last conclusion opens up a promising new research path. In non-relativistic quantum mechanics, the external fields acting on a system are not quantized, and this fact is what breaks down the harmony of the free case: the Schrödinger equation loses its Galilean invariance, and the Hamiltonian is no longer the generator of time-displacements in the Galilean group. In quantum field theory (QFT), on the contrary, fields are quantum items and not "external" fields affecting the behavior of the quantum system. As a consequence, the generators of the Poincaré group do not need to be reinterpreted in the presence of "external" factors, and the dynamical laws are always Poincaré-invariant. These features of QFT make us to consider whether the Actualization Rule, expressed in terms of the Casimir operators of the Galilean group in non-relativistic quantum mechanics, can be transferred to QFT by changing accordingly the symmetry group: the actual-valued observables of a system in QFT would be those represented by the Casimir operators of the Poincaré group, and the observables commuting with them and having, at least, the same symmetries. Since M and S^2 are the only Casimir operators of the Poincaré group, they would always be actual-valued observables. This conclusion would stand in agreement with a usual physical assumption in QFT: elemental particles always have actual values of mass and spin, and those values are precisely what define the different kinds of elemental particles of the theory.

Chapter 4

THE PHYSICAL RELEVANCE OF THE INTERPRETATION

As pointed out, during the last decades the discussions on the interpretation of quantum mechanics were based on the formal properties of the mathematical structure of the theory, and the traditional interpretative problems were analyzed from this perspective. But quantum mechanics is a physical theory and, as a consequence, a "good" interpretation of quantum mechanics should show its agreement with the orthodox practice of physics. In this chapter we shall argue for the physical relevance of our MHI by applying it to very well-known models and experimental results.

4.1. FREE POINTLIKE PARTICLE

The Hamiltonian of the free particle reads

$$H = \frac{P^2}{2m} = \frac{P_x^2 + P_y^2 + P_z^2}{2m} \tag{4-1}$$

where P is the momentum observable, with components P_x, P_y, P_z, and m is the mass of the particle. The particle is said to be "free" because there are not fields acting on it: then, space is homogeneous and, as a consequence, H is invariant under space-displacements in any direction (an analogous argument could be given in terms of the isotropy of space). The components

P_x, P_y, P_z are the generators of the symmetry and, at the same time, constants of motion of the system. Therefore, the Hamiltonian is degenerate.

According to our Actualization Rule, H acquires an actual value, and also P^2 since it is proportional to H and, then, has the same space-displacement symmetry (P^2 is the Casimir operator of the group generated by P_x, P_y, P_z). Nevertheless, P_x, P_y, P_z are not actual-valued because, being the generators of the symmetry, the actualization of any of their eigenvalues would break the symmetry of the free particle, in the sense of introducing an asymmetry non contained in the Hamiltonian.

Of course, the three components P can be used for the theoretical description of the free particle; in fact, usually any two of them are added to H to constitute a complete set of commuting observables (CSCO), $\{H, P_x, P_y\}$, $\{H, P_y, P_z\}$ or $\{H, P_x, P_z\}$, that defines a basis of the Hilbert space (given the functional dependence among the four magnitudes, the CSCO $\{P_x, P_y, P_z\}$ can be equivalently used). But this fact does not mean that those observables have to be considered actual-valued. On the contrary, the application of our MHI to this system agrees with the empirical non-accessibility to the values of P_x, P_y, P_z in the free particle. If we wanted to know these values, we would have to perform a measurement on the particle. But a measurement always involves an interaction with the measured object, which breaks the symmetry of the original system by modifying its Hamiltonian (for instance, consider a screen acting as a potential barrier that breaks the homogeneity of space). This means that, under measurement, the particle is no longer free: the symmetry breaking introduced by the interaction with the measuring apparatus is what allows us to have empirical access to an observable that was a symmetry generator of the original free system.

4.2. FREE PARTICLE WITH SPIN

The spin S is an internal contribution to the total angular momentum and, therefore, adds further degrees of freedom to the particle: the Hilbert space is now $\mathcal{H} = \mathcal{H}_f \otimes \mathcal{H}_s$, where $\mathcal{H}_f$ is the Hilbert space of the free particle and $\mathcal{H}_s$ is the Hilbert space of the spin. In this case, the Hamiltonian is

$$H = \frac{P^2}{2m} + E_0 \tag{4-2}$$

where E_0 can only be a multiple of S^2 and, then, may be conceived as an internal contribution to the energy (see Ballentine 1989).

According to our interpretation, in this case the system is composite, because it can be decomposed into two non-interacting subsystems (see the interpretative postulate DP): a free particle without spin, represented in $\mathcal{H}_f$ and with Hamiltonian $H_f = P^2/2m$, and a spin system, represented in $\mathcal{H}_s$ and with Hamiltonian $H_s = k\,S^2$, with $k = const.$ Then, the Actualization Rule has to be applied independently to each elemental subsystem.

The rule applies to the free particle subsystem as explained in the previous section. On the other hand, in the spin subsystem, H_s is invariant under space-rotation: the generators of this symmetry are the three components J_x, J_y, J_z of the total angular momentum J. But since in this case the orbital angular momentum L is zero, the total angular momentum $J = L + S$ turns out to be simply $J = S$, and the three components S_x, S_y, S_z of the spin S are the generators of the space-rotation symmetry. Analogously to the case of the free particle, according to our Actualization Rule, in this case H_s acquires an actual value, and also S^2 since it is proportional to H_s (S^2 is the Casimir operator of the group generated by S_x, S_y, S_z); nevertheless, S_x, S_y, S_z are not actual-valued since they are the generators of the symmetry of the Hamiltonian $H_s = k\,S^2$

Again, this conclusion agrees with the fact that we have no empirical access to the spin components of the free particle with spin. If we want to know the value of those components, we have to perform a measurement on the system: we have to introduce a magnetic field $\boldsymbol{B}$ of modulus B in some direction, say z, which breaks the isotropy of space and, as a consequence, the original space-rotation symmetry. Under the action of $\boldsymbol{B}$, the Hamiltonian H_s is not invariant under space-rotation anymore, because now it includes the interaction $-\gamma B S_z$ that privileges a particular direction of space. In other words, we can have experimental access to the spin component S_z only by means of a measurement that breaks the space-rotation symmetry of the original Hamiltonian and, therefore, makes the system no longer free. This is the usual way in which a spin component is measured in a Stern-Gerlach experiment (we shall analyze this measurement in detail in Chapter 5, Section 5.3).

4.3. HARMONIC OSCILLATOR

In general, an harmonic oscillator is an object affected by a quadratic potential energy, which produces a restoring force against displacement from equilibrium that is proportional to the displacement. Under a usual description, a quantum harmonic oscillator is a system of two bodies interacting through a potential quadratic in the relative displacement. The Hamiltonian of the system reads

$$H = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + k\left(Q_1 - Q_2\right)^2 \tag{4-3}$$

where k measures the strength of the interaction. Now we can define the coordinates of the center of mass and the relative coordinates,

$$Q_C = \frac{m_1 Q_1 + m_2 Q_2}{m_1 + m_2} \qquad Q_R = Q_1 - Q_2 \tag{4-4}$$

$$P_C = M\dot{Q}_C = P_1 + P_2 \qquad P_R = \mu\dot{Q}_R = \frac{m_1 P_1 - m_2 P_2}{m_1 + m_2} \tag{4-5}$$

where $M = m_1 + m_2$ is the total mass, and $\mu = m_1 m_2 / \left(m_1 + m_2\right)$ is the reduced mass. In this new coordinate system, the Hamiltonian can be written as

$$H = \frac{P_C^2}{2M} + \frac{P_R^2}{2\mu} + kQ_R^2 \tag{4-6}$$

Then, H can be expressed as the sum of two terms, $H = H_0 + H_K$, such that

$$H_0 = W = H - \frac{P_C^2}{2M} = \frac{P_R^2}{2\mu} + kQ_R^2 \qquad H_K = \frac{P_C^2}{2M} \tag{4-7}$$

It is quite clear that H_0 and H_K commute, since H_0 only depends on the relative coordinates and H_K only depends on the coordinates of the center of mass. Therefore, the total Hamiltonian can be written as (for notation simplicity, from now on we shall ignore the difference between H_K and H_K^P and between H_0 and H_W^Q as introduced in Chapter 3, Section 3.3)

$$\left[H_0, H_K\right] = 0 \quad \Rightarrow \quad H = H_0 \otimes I^P + I^Q \otimes H_K \tag{4-8}$$

In this case it is easy to see that H is the Hamiltonian of a composite system $\mathcal{S}$, whose subsystems $\mathcal{S}_0$ with Hamiltonian H_0 and $\mathcal{S}_K$ with Hamiltonian H_K do not interact with each other; thus, they "actualize" independently. This means that H_0 (internal energy) acquires an actual value in $\mathcal{S}_0$, and H_K (kinetic energy) acquires an actual value in $\mathcal{S}_K$. In the system $\mathcal{S}_0$, the Hamiltonian H_0 can be expressed in terms of the dimensionless position and momentum operators, $q = \left(2k/\hbar\Omega\right)^{1/2} Q_R$ and $p = \left(1/\mu\hbar\Omega\right)^{1/2} P_R$, where Ω is the frequency of oscillation,

$$H_0 = W = \frac{P_R^2}{2\mu} + kQ_R^2 = \frac{1}{2}\hbar\Omega\left(p^2 + q^2\right) \tag{4-9}$$

In turn, if the observable number of modes $N = a^\dagger a$ is used,

$$N = a^\dagger a = \left(\frac{q - ip}{\sqrt{2}}\right)\left(\frac{q + ip}{\sqrt{2}}\right) \Rightarrow H_0 = \hbar\,\Omega\left(N + \frac{1}{2}\right) \tag{4-10}$$

As it is well known, in this case the spectra of H and N can be obtained algebraically:

$$H_0\left|n\right\rangle = \omega_n\left|n\right\rangle \tag{4-11}$$

$$N\left|n\right\rangle = n\left|n\right\rangle \tag{4-12}$$

In fact, since H_0 has no symmetries, it is non-degenerate: the CSCO $\{H_0\}$ defines a basis of the Hilbert space of the system. According to our Actualization Rule, H_0 acquires an actual value and, due to its non-

degeneracy, the preferred context that it defines corresponds to the basis $\{|n\rangle\}$: any observable commuting with H_0 (that is, whose eigenvectors are vectors of $\{|n\rangle\}$) is also actual-valued. In particular, the number of particles N acquires an actual value since $[N, H_0] = 0$.

The harmonic oscillator has a central relevance in quantum mechanics because it provides a model for may kinds of vibrating systems. In particular, the electromagnetic field can be decomposed in terms of linearly independent modes, each one of which behaves as an harmonic oscillator usually associated to a particle; in this case, N is conceived as the observable number of particles. But the point to stress here is that, in all of those vibrating phenomena, the energy of the system is the relevant physical magnitude, whose values are experimentally accessible, and our Actualization Rule accounts for this fact.

4.4. FREE HYDROGEN ATOM

The hydrogen atom is conceived as a two-body system consisting of an electron and a proton interacting with each other through a Coulombian potential. In this case, the Hamiltonian reads

$$H = \frac{P_e^2}{2m_e} + \frac{P_p^2}{2m_p} - \frac{e^2}{|Q_e - Q_p|} \tag{4-13}$$

where the subindexes e and p refer to the electron and the proton respectively, and e is the electric charge of the electron. As usual, if we take the center of mass coordinates and the relative coordinates as independent variables, we obtain eqs. (4-4) and (4-5), with the indexes e and p instead of 1 and 2 respectively. Then, in the new coordinates, the total Hamiltonian results

$$H = \frac{P_C^2}{2M} + \frac{P_R^2}{2\mu} - \frac{e^2}{|Q_R|} \tag{4-14}$$

where we can identify H_0 and H_K as

$$H_0 = W = H - \frac{P_C^2}{2M} = \frac{P_R^2}{2\mu} - \frac{e^2}{|Q_R|} \qquad H_K = \frac{P_C^2}{2M} \tag{4-15}$$

Here it is also clear that

$$\left[H_0, H_K\right] = 0 \quad \Rightarrow \quad H = H_0 \otimes I^P + I^Q \otimes H_K \tag{4-16}$$

Again, the hydrogen atom is a composite system that can be analyzed into a subsystem $\mathcal{S}_0$, defined by the internal energy $H_0 = W$, and a subsystem $\mathcal{S}_K$, defined by the kinetic energy H_K. And, according to the modal-Hamiltonian Actualization Rule, both subsystems "actualize" independently.

The usual strategy for solving the energy eigenvalue equation consists in taking the reference fame at rest with respect to the center of mass of the system, in such a way that $P_C = H_K = 0$ and $H = H_0 = W$. As it is well known, when the resulting equation is written in spherical coordinates (r,θ,ϕ), its solution can be expressed as the product of two functions, one only dependent on the radial coordinate and the other only dependent on the angular coordinates: $\Psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$. By solving the radial and the angular equations, three "good" quantum numbers are obtained: the principal quantum number n, the orbital angular momentum quantum number l and the magnetic quantum number m_l. These quantum numbers correspond to the eigenvalues of the observables H, L^2 and L_z respectively, where L is the orbital angular momentum, and L_x, L_y, L_z are its components:

$$H\left|n,l,m_l\right\rangle = \omega_n \left|n,l,m_l\right\rangle \tag{4-17}$$

$$L^2\left|n,l,m_l\right\rangle = l(l+1)\hbar^2 \left|n,l,m_l\right\rangle \tag{4-18}$$

$$L_z\left|n,l,m_l\right\rangle = m_l\hbar \left|n,l,m_l\right\rangle \tag{4-19}$$

with $n = 0,1,2,\ldots$, $l < n$, and $-l \le m_l \le l$. In particular, the energy eigenvalues are computed as

$$\omega_n = -\frac{\mu e^4}{2\hbar^2 n^2} \tag{4-20}$$

Therefore, the hydrogen atom is described in terms of the basis $\{|n,l,m_l\rangle\}$ defined by the CSCO $\{H, L^2, L_z\}$: the quantum numbers n, l, and m_l label the solutions Ψ_{nlm_l} of the energy eigenvalue equation.

In this case, the Hamiltonian is degenerate due to its space-rotation invariance. When the spin of the electron is not considered (for the effect of the spin, see below, Chapter 4, Section 4.6), the total angular momentum $J = L + S$ is simply $J = L$. Then, the three components L_x, L_y, L_z of L are the generators of the symmetry group, and L^2 is the Casimir operator of the group. As a consequence, although l, and m_l are good quantum numbers in the sense of collaborating in the definition of a basis of the Hilbert space, the eigenvalues ω_n of the Hamiltonian do not depend on them: due to the symmetry of H, the values of L^2 and L_z have no manifestations in the energy spectrum. According to our Actualization Rule, as the result of the degeneracy of H, the observables L^2 and L_z do not acquire actual values: the only actual-valued observables of the system are H and the observables having, at least, the same space-rotation symmetry (at least, the same degeneracy) as H.

The fact that our MHI does not confer actual values to L^2 and L_z should agree with experimental evidence, in particular, with the data coming from spectroscopy. Let us consider each observable in detail:

a) In quantum chemistry, the states Ψ_{nlm_l} of the atom (orbitals) are labeled as $X\alpha$, where X is the principal quantum number n, and α is replaced with s, p, d, f, etc., that is, with letters corresponding to the value of the angular momentum quantum number l: $1s$: $2s$, $2p$, $3s$, $3p$, $3d$, etc. As we can see, the magnetic quantum number m_l is not included in those labels because, although Ψ_{nlm_l} depends on the three quantum numbers, the space-rotation symmetry of the Hamiltonian makes the selection of L_z a completely arbitrary decision: since space is isotropic, we can choose L_x or L_y to obtain an equally legitimate description of the free atom. The arbitrariness in the selection of the z-direction is manifested in spectroscopy by the fact that the spectral lines give no experimental evidence about the values of L_z: we have no empirical access to the number m_l. Our interpretation, that does not assign an actual value to L_z, agrees with those experimental results. Analogously to the case of the free particle with spin (Section 4.2), if we want to know the value of L_z, we

have to introduce a magnetic field that breaks the isotropy of space (we shall describe this situation in detail in the next section).

b) On the contrary, the value of the quantum number l is included in the traditional orbitals' labels as s, p, d, etc. Moreover, the value of l can be inferred from the observed energy spectrum of the hydrogen atom, and it plays a role in the explanation of the well-known spectral series (Lyman, Balmer, Paschen, etc.). These facts might be interpreted as a symptom of the actual-valuedness of L^2 in the free hydrogen atom. However, the manifestation of the value of l requires the interaction between the atom and an electromagnetic field. The usual explanation runs as follows. Since energy transitions involve the absorption or emission of a photon (spin 1), conservation of the angular momentum forces the atom to experience a change of 1 in its orbital angular momentum L. For this reason, when a photon is absorbed by an atom in an s orbital, the atom acquires orbital momentum and makes a transition to a p orbital; when absorbed by an atom in a p orbital, the orbital momentum increases ($p \rightarrow d$ transition) or decreases ($p \rightarrow s$ transition), depending on the relative orientations of the photon and the atom angular momenta. But transitions $s \rightarrow d$ or $p \rightarrow f$ are forbidden. From this explanation, it is clear that the manifestation of the value of l is the result of an interaction; but, then, the system is not the free hydrogen atom anymore. The new system has a Hamiltonian of the form

$$H = H_{at} + H_{em} + H_{\text{int}} \tag{4-21}$$

where H_{at} is the Hamiltonian of the free hydrogen atom (see eq. (4-13)), and H_{em} is the Hamiltonian of the electromagnetic field, which can be computed as the infinite sum of the Hamiltonians of the independent harmonic oscillators corresponding to the infinite modes of the field (see eq. (4-5)). In turn, H_{int} is the interaction Hamiltonian, that depends on the dipole moment of the atom and on the electric field (see Ballentine 1998, pp. 548-549). The interaction breaks the original symmetry in L^2 and, as a consequence, removes the energy degeneracy in the quantum number l: now the energy eigenvalues ω_{nl} turn out to be functions of both the quantum numbers n and l. This fact is what leads to the manifestation of

the value of l in the energy spectrum, and allows L^2 to become an actual-valued observable in the new, non-free system.

The fact that L^2 is not an actual-valued observable in the free *hydrogen* atom does not mean that it never acquires an actual value in a free atom. The particular features of the hydrogen atom strongly depend on the Coulombian potential, conceived as generated by its one-proton nucleus. In more complex atoms, the potential in not perfectly Coulombian, and this asymmetry removes the degeneracy in l of the Hamiltonian: the energy eigenvalues ω_{nl} are functions of both n and l with no need of interaction (see Ballentine 1998, p. 280). This means that L^2 does no longer discriminate among the different eigenvectors corresponding to a single degenerate energy eigenvalue, but rather removes the degeneracy of the symmetric Coulombian case. According to our Actualization Rule, this implies that L^2 is an actual-valued observable for free atoms with non-Coulombian potential.

4.5. ZEEMAN EFFECT

When an external magnetic field is applied to the atom, the spectral lines split into multiple closely spaced lines. First observed by Pieter Zeeman in 1896, this phenomenon is known as Zeeman effect.

In the previous section we have seen that, either in the Coulombian or in the non-Coulombian potential case, the Hamiltonian is endowed with a space-rotation symmetry that makes the energy eigenvalues to be independent of the magnetic quantum number m_l, that is, to be degenerate in m_l. It is precisely due to this symmetry that the selection of L_z for completing the basis of the Hilbert space is the result of an arbitrary decision. The arbitrariness of choosing the z-direction agrees with the fact that there is no experimental evidence about the value of m_l in the energy spectrum.

In the case of the Zeeman effect, the magnetic field breaks the isotropy of space. In this case, the general Lagrangian of the system of two particles (one proton and one electron) in a magnetic field is

$$\mathcal{L} = \frac{1}{2} m_1 \dot{Q}_1^2 + \frac{1}{2} m_2 \dot{Q}_2^2 + |e| \boldsymbol{A} \cdot \dot{Q}_1 - |e| \boldsymbol{A} \cdot \dot{Q}_2 - V\left(|Q_1 - Q_2|\right) \qquad (4\text{-}22)$$

where Q_1 ($\dot{Q}_1$) is the position (velocity) of the proton, Q_2 ($\dot{Q}_2$) is the position (velocity) of the electron, and $\boldsymbol{A}$ is the vector potential depending only on the coordinates. The opposite signs of the third and fourth terms in (4.22) comes from the opposite signs of the charges of the proton and the electron.

Now we introduce the same change of coordinates as in the case of the harmonic oscillator (see eqs. (4-4) and (4-5)). Then, the Lagrangian reads

$$\mathcal{L} = \frac{1}{2} M \dot{Q}_C^2 + \frac{m_1 m_2}{M} \dot{Q}_R^2 + |e| \boldsymbol{A} \cdot \dot{Q}_R - V\left(|Q_R|\right) \tag{4-23}$$

where $M = m_1 + m_2$. In order to obtain the Hamiltonian description of the system, we have to compute the momenta of the coordinates by means of the definitions $P_C = \partial \mathcal{L} / \partial \dot{Q}_C$ and $P_R = \partial \mathcal{L} / \partial \dot{Q}_R$. Then, the Hamiltonian $H = P_C \dot{Q}_C + P_R \dot{Q}_R - \mathcal{L}$ reads

$$H = \frac{P_C^2}{2M} + \frac{M}{m_1 m_2}\left(P_R - |e| \boldsymbol{A}\right)^2 + V\left(|Q_R|\right) \tag{4-24}$$

If the magnetic field is uniform, the vector potential can be written as $\boldsymbol{A} = (1/2) Q_R \times \boldsymbol{B}$. Then, $P_R \cdot \boldsymbol{A}$ can be computed as

$$P_R \cdot \boldsymbol{A} = -\frac{1}{2} P_R \cdot \left(P_R \times \boldsymbol{B}\right) = -\frac{1}{2} Q_R \times P_R \cdot \boldsymbol{B} = -\frac{1}{2} L \cdot \boldsymbol{B} \tag{4-25}$$

Therefore, the Hamiltonian can be written as

$$H = \frac{P_C^2}{2M} + \frac{P_R^2}{2\mu} + V\left(|Q_R|\right) - \frac{\mu_B}{\hbar} L \cdot \boldsymbol{B} - \frac{|e|^2}{8\mu}\left(Q_R \times \boldsymbol{B}\right)^2 \tag{4-26}$$

where $\mu = m_1 m_2 / M$, and $\mu_B = e\hbar / 2\mu$ is the Bohr magneton. In this Hamiltonian we can immediately identify two commuting Hamiltonians H_R and H_C,

$$H_R = \frac{P_R^2}{2\mu} + V\left(|Q_R|\right) - \frac{\mu_B}{\hbar} L \cdot \boldsymbol{B} - \frac{|e|^2}{8\mu}\left(Q_R \times \boldsymbol{B}\right)^2$$
$$H_C = \frac{P_C^2}{2M} \tag{4-27}$$

in such a way that the internal energy of the composite system is

$$W = H - H_C = H_R = \\ = \frac{P_R^2}{2\mu} + V\left(|Q_R|\right) - \frac{\mu_B}{\hbar} L \cdot \boldsymbol{B} - \frac{|e|^2}{8\mu}\left(Q_R \times \boldsymbol{B}\right)^2 \tag{4-28}$$

In this case, in spite of the action of the external field, the total system $\mathcal{S}$ is composite: the subsystem $\mathcal{S}_C$, with Hamiltonian H_C, carries the total kinetic energy of translation; the Hamiltonian of the subsystem $\mathcal{S}_R$ is the internal energy $W = H_R$, and the set of actual-valued observables of $\mathcal{S}_R$ is independent of the kinetic energy and, therefore, invariant under boost-transformations. In general, the term $\left(|e|^2/8\mu\right)\left(Q_R \times \boldsymbol{B}\right)^2 = \left(|e|^2/2\mu\right)\boldsymbol{A}^2$ is neglected, and the Hamiltonian is written as (see Ballentine 1998, p. 325; Cohen-Tannoudji, Diu and Lalöe 1977, p. 835)

$$H = \frac{P_C^2}{2M} + \frac{P_R^2}{2\mu} + V\left(|Q_R|\right) - \frac{\mu_B}{\hbar} L \cdot \boldsymbol{B} \tag{4-29}$$

Again, when the system is described in the reference frame at rest with respect to the center of mass, $P_C = H_C = 0$ and $H = H_R = W$:

$$H = \frac{P_R^2}{2\mu} + V\left(|Q_R|\right) - \frac{\mu_B}{\hbar} L \cdot \boldsymbol{B} = H_{at} - \frac{\mu_B}{\hbar} L \cdot \boldsymbol{B} \tag{4-30}$$

This means that the magnetic field $\boldsymbol{B}$ along the z-axis breaks the isotropy of space and, as a consequence, the space-rotation symmetry of the Hamiltonian. In turn, the breaking of the symmetry removes the energy degeneracy in m_l: now L_z is not arbitrarily chosen but selected by the direction of the magnetic field. As a consequence, the original degeneracy of the $(2l+1)$-fold multiplet

of fixed n and l is now removed: the energy levels turn out to be displaced by an amount

$$\Delta\omega_{nlm_l} = \frac{|e|\hbar B}{2m_e c} m_l \tag{4-31}$$

This means that the Hamiltonian, with eigenvalues ω_{nlm_l}, is now non-degenerate: it constitutes by itself the CSCO $\{H\}$ that defines the preferred basis $\{|n,l,m_l\rangle\}$. According to our Actualization Rule, in this case H and all the observables commuting with H are actual-valued: since this is the case for L^2 and L_z, both observables acquire actual values in agreement with the experimental evidence of the Zeeman effect.

4.6. FINE STRUCTURE

When the spectral lines of the hydrogen atom corresponding to $n > 1$ are examined at a very high resolution, they are found to be closely spaced doublets. This splitting was one of the first experimental evidences of the electron spin. This phenomenon is usually explained by saying that the energy levels of the atom are affected by the interaction between the electron spin S and the orbital angular momentum L. Now the Hamiltonian of the system reads

$$H = H_{at} + H_s + H_{s-o} \tag{4-32}$$

where H_{at} is again the Hamiltonian of the free hydrogen atom, $H_s = k S^2$ is the Hamiltonian of the spin, and H_{s-o} is the Hamiltonian representing the spin-orbit interaction, function of the product $L \cdot S$.

When the spin-orbit interaction is neglected ($H_{s-o} = 0$), the system is composite and can be described in terms of the basis $\{|n,l,m_l,s,m_s\rangle = |n,l,m_l\rangle \otimes |s,m_s\rangle\}$, where the $s(s+1)\hbar^2$ are the eigenvalues of S^2, and the $m_s\hbar$ are the eigenvalues of S_z. But when the spin-orbit interaction is taken into account, the observables L_z and S_z no longer commute with H and, therefore, they are not constants of motion of the system: it is usually said that m_l and m_s are not good quantum numbers

anymore. Nevertheless, the Hamiltonian is still invariant under space-rotation: the components J_x, J_y, J_z of the total angular momentum J are the generators of the symmetry group, and J^2 is the Casimir operator of the group, with eigenvalues $j(j+1)\hbar^2$. In turn, J is the sum of the orbital angular momentum L and the spin angular momentum S:

$$J = L + S \quad m_j = m_l + m_s \tag{4-33}$$

where m_j corresponds to the eigenvalue of J_z. So, now m_j is a good quantum number. But we also know that

$$J^2 = \left(L + S\right)^2 \Rightarrow L \cdot S = \frac{J^2 - L^2 - S^2}{2} \tag{4-34}$$

Therefore, H_{s-o} is a function of J^2, L^2 and S^2, and the corresponding quantum numbers j, l and s are also good quantum numbers. As a consequence, the eigenvalues of the total Hamiltonian have the general form

$$\omega_{nljs} = \omega_{nl} + \xi(nl)\left[\, j\left(j+1\right) - l\left(l+1\right) - s\left(s+1\right)\right] \tag{4-35}$$

where the ω_{nl} represent the energy eigenvalues with no spin-orbit coupling, and ξ is a function of nl (see Tinkham 1964, pp. 181-183). Then, the basis $\left\{\left|n, l, j, s, m_j\right\rangle\right\}$ of the Hilbert space of the system is defined by the CSCO $\left\{H, L^2, J^2, S^2, J_z\right\}$, where

$$H\left|n, l, j, s, m_j\right\rangle = \omega_{nljs}\left|n, l, j, s, m_j\right\rangle \tag{4-36}$$

$$L^2\left|n, l, j, s, m_j\right\rangle = l(l+1)\hbar^2\left|n, l, j, s, m_j\right\rangle \tag{4-37}$$

$$J^2\left|n, l, j, s, m_j\right\rangle = j(j+1)\hbar^2\left|n, l, j, s, m_j\right\rangle \tag{4-38}$$

$$S^2\left|n, l, j, s, m_j\right\rangle = s(s+1)\hbar^2\left|n, l, j, s, m_j\right\rangle \tag{4-39}$$

$$J_z\left|n, l, j, s, m_j\right\rangle = m_j\hbar\left|n, l, j, s, m_j\right\rangle \tag{4-40}$$

It is quite clear that the spin-orbit coupling removes the original degeneracy of the eigenvalues ω_{nl} of the atom with no coupling. Therefore, in this case our Actualization Rule selects L^2, J^2 and S^2 as actual-valued observables, because all of them commute with H and have the same degeneracy in m_j as H. But the space-rotation symmetry still present in the system leads to a degeneracy of H, manifested by the fact that the energy eigenvalues ω_{nljs} do not depend on m_j. Then, according to our Actualization Rule, although in this case m_j is a good quantum number, J_z does not acquire an actual value, and this result agrees with the arbitrariness of the selection of the z-direction for J_z.

When a magnetic field is applied to the atom, the spectral lines split in different ways. The "normal" Zeeman effect, explained in the previous section, is observed in spin 0 states where, obviously, the spin-orbit coupling has no effect. In the states where the spin-orbit coupling is effective, the action of the magnetic field produces a further splitting of the energy levels known as "anomalous" Zeeman effect. Nevertheless, the explanation of the anomalous effect is the same as that of the normal effect: the action of the magnetic field along the z-axis breaks the space-rotation symmetry of the Hamiltonian by privileging the z-direction, and this leads to the removal of the original degeneracy of the Hamiltonian in the quantum number m_j (instead of in the quantum number m_l as in the normal effect). In this case, our Actualization Rule prescribes that J_z will be also actual-valued.

4.7. THE BORN-OPPENHEIMER APPROXIMATION

Our Actualization Rule endows the Hamiltonian of the system with the role of selecting the preferred context and, therefore, the energy of the system always acquires an actual value. But this does not mean that the momentum is an actual-valued observable in any case, since it does not always commute with the Hamiltonian. In fact, when a system is not affected by a vector field, its Hamiltonian has the general form

$$H = \frac{P^2}{2m} + V(Q) \tag{4-41}$$

When the mass m of the system is small, the kinetic term prevails over the potential term, and the Hamiltonian approximately commutes with P^2. In turn, for very large masses, the kinetic term can be neglected and H approximately commutes with $V(Q)$. So, the modal-Hamiltonian Actualization Rule supports the usual claim that "small" systems approximately actualize in momentum and "large" systems approximately actualize in position. In this sense, the MHI agrees with the physical assumption that electrons have definite momentum but not definite position, and the nucleus has definite position but not definite momentum. In general, our rule explains the fact that macroscopic systems, with their large masses, –approximately– posses a definite value of position.

This point has a particular relevance in molecular chemistry, where the description of molecules is based on the adiabatic separation of electron and nuclear motions. As it is well known, the Born-Oppenheimer approximation conceives the nuclei as classical-like particles, that is, as precisely localized objects. This approximation strategy of holding the nucleus at rest in an actual position can be thought off as formally arising from making the masses of the nuclei infinite. However, from a strictly quantum-mechanical viewpoint, without a rule for selecting the actual-valued observables of the system, the assumption of infinite nuclear masses does not explain yet why the nucleus can be treated as having an actual value of position. As Primas says, "*we hardly understand why the Born-Oppenheimer* picture *is compatible with the* concepts *of quantum mechanics*" (Primas 1983, p. 13; see also Woolley 1978, Amann 1992).

In fact, the total Hamiltonian of a molecule reads

$$H_{tot} = T_n(P_\alpha) + V_{nn}(R_\alpha) + T_e(p_i) + V_{ee}(r_i) + V_{en}(r_i, R_\alpha) \qquad (4\text{-}42)$$

where T_n is the nuclear kinetic energy (function of the nuclear momenta P_α), V_{nn} is the potential due to the interactions between the nuclei (function of the nuclear positions R_α), T_e is the electronic kinetic energy (function of the electronic momenta p_i), V_{ee} is the potential due to the interactions between the electrons (function of the electronic positions r_i), and V_{en} is the potential due to the interactions between the electrons and the nuclei (function of the r_i and the R_α). The Born-Oppenheimer approximation proceeds in two steps.

a) In the first step the nuclear kinetic energy is neglected, that is, $T_n\left(P_\alpha\right)$ is substracted from the total Hamiltonian H_{tot}. The resulting electronic Hamiltonian H_e reads

$$H_e = V_{nn}\left(R_\alpha\right) + T_e\left(p_i\right) + V_{ee}\left(r_i\right) + V_{en}\left(r_i, R_\alpha\right) \tag{4-43}$$

where the nuclear positions R_α play the role of parameters. Therefore, the nuclear potential $V_{nn}\left(R_\alpha\right)$ is just a constant that shifts the eigenvalues of H_e only by some constant amount. Thus, the electronic Schrödinger equation

$$H_e \Psi_e\left(r_i; R_\alpha\right) = E_e\left(R_\alpha\right) \Psi_e\left(r_i; R_\alpha\right) \tag{4-44}$$

is solved, and the electronic energy eigenvalues $E_e\left(R_\alpha\right)$, parametrically depending on the R_α, are obtained. This step is often referred to as the *clamped nuclei approximation*: the electron-nucleus interactions, represented by $V_{en}\left(r_i, R_\alpha\right)$, are conceived in terms of electrons in the Coulomb potential produced by nuclei "clamped" at definite positions. The substraction of $T_n\left(P_\alpha\right)$ is justified by assuming that $T_n \ll T_e$, which, in turn, relies on the assumption that the nuclear mass is much greater than the electronic mass: $M \gg m$. In particular, if $M/m \to \infty$, then $T_n\left(P_\alpha\right) \to 0$.

b) In the second step the nuclear kinetic energy is reintroduced and the total energy E_{tot} is obtained by solving the nuclear Schrödinger equation,

$$\left[T_n\left(P_\alpha\right) + E_e\left(R_\alpha\right)\right] \Psi_n\left(R_\alpha\right) = E_{tot} \Psi_n\left(R_\alpha\right) \tag{4-45}$$

It is clear that the crucial approximation of the Born-Oppenheimer strategy is introduced in the first step, where the relation $M \gg m$ is approximated to $M/m \to \infty$. Of course, this limit is never strictly true and, therefore, the results so obtained are mere approximations. But this is not the point here. Let us suppose for a moment that the nuclear mass were effectively infinite; according to the usual reading of the Born-Oppenheimer approximation, in this case we could infer that the nuclei are effectively clamped at definite positions. The question is why we can make this inference.

The uncritical answer relies on intuitions coming from classical physics: a body with infinite mass M would have null kinetic energy $T = P^2 / 2M$ and, as a consequence, it would be at rest in a definite position. However, here we are not in a classical domain, but in a quantum theoretical framework where, as it is well known, classical intuitions usually do not work.

The usual quantum answer is more adequate than the previous one, because it relies on quantum concepts. Let us recall that the total Hamiltonian H_{tot} of the molecule can be expressed as (see eqs. (4-42) and (4-43))

$$H_{tot} = T_n\left(P_\alpha\right) + H_e\left(p_i, r_i, R_\alpha\right) \tag{4-46}$$

where the electronic Hamiltonian H_e is not a function of the nuclear momenta P_α. When $M/m \to \infty$ and, then, $T_n\left(P_\alpha\right) \to 0$, both Hamiltonians can be considered as equal, $H_{tot} = H_e\left(p_i, r_i, R_\alpha\right)$. As a consequence, since R_α commutes with H_e, when $M/m \to \infty$ R_α also commutes with H_{tot}:

$$H_{tot} = H_e\left(p_i, r_i, R_\alpha\right) \quad \Rightarrow \quad \left[H_{tot}, R_\alpha\right] = 0 \tag{4-47}$$

On this basis, the quantum answer reads as follows: since the molecule is in a time-independent state (essentially an axiom of quantum chemistry), its state is an eigenvector of H_{tot} and, as a consequence, it has an actual value of H_{tot} (that is, of its energy). Since the molecule has an actual value of its H_{tot}, then the R_α, commuting with H_{tot}, are also actual-valued, and this means that the nuclei are located at actual positions.

Although seemingly reasonable, this explanation takes for granted the actual value of H_{tot}, a point that is far from being clear from a quantum-mechanical viewpoint. When the quantum answer is analyzed with care, it is not difficult to see that it is implicitly based on the traditional eigenstate-eigenvalue link, according to which, when a quantum system is in a state $\left|\varphi\right\rangle$, an observable A is actual-valued iff $\left|\varphi\right\rangle$ is an eigenvector of A. Now the quantum answer turns out to be a precise argument:

– Since the molecule is in a stationary state, its state $\left|\Psi\right\rangle$ is an eigenvector of H_{tot}.

– Since $|\Psi\rangle$ is an eigenvector of H_{tot}, according to the eigenstate-eigenvalue link, H_{tot} is actual-valued.

– Since $M/m \to \infty$, then $[H_{tot}, R_\alpha] = 0$

– Since $|\Psi\rangle$ is an eigenvector of H_{tot} and $[H_{tot}, R_\alpha] = 0$, then $|\Psi\rangle$ is an eigenvector of R_α.

– Since $|\Psi\rangle$ is an eigenvector of R_α, according to the eigenstate-eigenvalue link, R_α is actual-valued, that is, the nuclei are "clamped" at definite positions.

The problem with this argument is that the eigenstate-eigenvalue link does not always work as well as one would expect: its shortcomings come to the light already in the simplest atomic model. In fact, in the free hydrogen atom each vector $|\Psi_{nlm_l}\rangle$ is an eigenvector of the three observables of the CSCO $\{H, L^2, L_z\}$. Therefore, according to the eigenstate-eigenvalue link, the three observables H, L^2 and L_z should be actual-valued. However, as we have seen, the space-rotation symmetry of the system makes the selection of the spatial direction z a completely arbitrary decision: since space is isotropic, we could choose a different direction z' to obtain an equally legitimate description of the free atom. In other words, the CSCO's $\{H, L^2, L_z\}$ and $\{H, L^2, L_{z'}\}$ supply both equally "good" descriptions of the free hydrogen atom. Therefore, according to the eigenstate-eigenvalue link, the observable $L_{z'}$ should also be actual-valued. But $[L_z, L_{z'}] \neq 0$: two non-commuting observables cannot have both actual values. On the other hand, if one of them were selected as the actual-valued observable, a physical fact would depend on an arbitrary descriptive decision; but this move would be unacceptable from a scientific viewpoint.

Our MHI provides an answer to this conceptual problem. For large masses, the Hamiltonian is –approximately– invariant under boost transformation and, therefore, it approximately commutes with position. As a consequence, according to the Actualization Rule, the position observable acquires an actual value: this provides a conceptual justification to the Born-Oppenheimer assumption. Of course, masses are never infinite: this is what makes the Born-Oppenheimer strategy an approximation and not a precise method. But also in this sense our interpretation agrees with the usual assumption: since the Hamiltonian perfectly commutes with position only in the infinite mass limit, only in this limit we can say with absolute precision

that position acquires an actual value. In real situations, the actual-valued observable will generally be an observable very “similar” to position, but which becomes indistinguishable from position for increasing masses.

Chapter 5

THE MEASUREMENT PROBLEM

In the standard von Neumann model, a quantum measurement is conceived as an interaction between a system $\mathcal{S}$ and a measuring device $\mathcal{D}$. Before the interaction, $\mathcal{D}$ is prepared in a ready-to-measure state $|r_0\rangle$, eigenvector of the pointer observable R of $\mathcal{D}$, and the state of $\mathcal{S}$ is a superposition of the eigenstates $|a_i\rangle$ of an observable A of $\mathcal{S}$. The interaction introduces a correlation between the eigenstates $|a_i\rangle$ of A and the eigenstates $|r_i\rangle$ of R:

$$|\psi_0\rangle = \sum_i c_i |a_i\rangle \otimes |r_0\rangle \quad \rightarrow \quad |\psi\rangle = \sum_i c_i |a_i\rangle \otimes |r_i\rangle \tag{5-1}$$

The problem consists in explaining why, being the state $|\psi\rangle$ a superposition of the $|a_i\rangle \otimes |r_i\rangle$, the pointer R acquires a definite actual value.

In the orthodox collapse interpretation, the pure state $|\psi\rangle$ is assumed to "collapse" to a mixture ρ^c:

$$\rho^c = \sum_i |c_i|^2 |a_i\rangle \otimes |r_i\rangle\langle a_i| \otimes \langle r_i| \tag{5-2}$$

where the probabilities $|c_i|^2$ are given an ignorance interpretation. Then, in this situation it is supposed that the measuring apparatus is in one of the eigenvectors $|r_i\rangle$ of R, say $|r_k\rangle$, and therefore R acquires a definite actual value r_k, the eigenvalue corresponding to the eigenvector $|r_k\rangle$, with probability $|c_k|^2$. In the modal interpretations, the problem is to explain the

definite reading of the pointer with its associated probability, without assuming the collapse hypothesis. In our MHI, the Actualization Rule is what must accomplish this task.

5.1. IDEAL MEASUREMENT

In the von Neumann model and, in general, in the discussions about the quantum measurement problem, the Hamiltonians involved in the process are usually not taken into account. In our interpretation, where the Hamiltonians plays a central role, we have to provide a more detailed model of the measurement process. Thus, we shall say that a quantum measurement is a three-stage process: (i) during Stage I ($t \leq 0$), the system $\mathcal{S}$ and the device $\mathcal{D}$ do not interact, (ii) during Stage II ($0 < t < t_1$), $\mathcal{S}$ and $\mathcal{D}$ interact, and the interaction establishes the correlation, and (iii) the interaction ends at $t = t_1$, and during Stage III ($t \geq t_1$), $\mathcal{S}$ and $\mathcal{D}$ do not interact.

Stage I: Let us suppose that we want to obtain the coefficients of the state $|\psi_S(t_0 = 0)\rangle \in \mathcal{H}_S$ of the elemental quantum system $\mathcal{S}$: $(\mathcal{O}_S = \mathcal{H}_S \otimes \mathcal{H}_S\,, H_S)$:

$$|\psi_S(t_0 = 0)\rangle = \sum_i c_i |a_i\rangle \in \mathcal{H}_S \tag{5-3}$$

where

$$A|a_i\rangle = a_i |a_i\rangle \in \mathcal{O}_S\,, \text{ where } \{|a_i\rangle\} \text{ is a basis of } \mathcal{H}_S \tag{5-4}$$

For simplicity, we shall assume that the Hamiltonian $H_S \in \mathcal{O}_S$ of S is non-degenerate:

$$H_S |\omega_{Si}\rangle = \omega_{Si} |\omega_{Si}\rangle\,, \text{ where } \{|\omega_{Si}\rangle\} \text{ is a basis of } \mathcal{H}_S \tag{5-5}$$

The measuring device is an elemental quantum system $\mathcal{D}$: $(\mathcal{O}_D = \mathcal{H}_D \otimes \mathcal{H}_D\,, H_D)$ having an observable $R \in \mathcal{O}_D$, which has to

possess different and macroscopically distinguishable eigenvalues in order to play the role of the pointer:

$$R\left|r_i\right\rangle = r_i\left|r_i\right\rangle, \text{ where } \left\{\left|r_i\right\rangle\right\} \text{ is a basis of } H_D \tag{5-6}$$

At time $t_0 = 0$, the device $\mathcal{D}$ is prepared in a ready-to-measure state $\left|r_0\right\rangle$, eigenvector of R:

$$\left|\psi_D(t_0)\right\rangle = \left|r_0\right\rangle \in H_D \tag{5-7}$$

For simplicity, we shall assume that the Hamiltonian $H_D \in \mathcal{O}_D$ of $\mathcal{D}$ is non-degenerate:

$$H_D\left|\omega_{Di}\right\rangle = \omega_{Di}\left|\omega_{Di}\right\rangle, \text{ where } \left\{\left|\omega_{Di}\right\rangle\right\} \text{ is a basis of } \mathcal{H}_D \tag{5-8}$$

For the reading of the pointer to be possible, the eigenvectors $\left|r_i\right\rangle$ of R have to be stationary. Thus, $\mathcal{D}$ is constructed in such a way that R commutes with H_D:

$$\left[H_D, R\right] = 0 \quad \Rightarrow \quad \left|\omega_{Di}\right\rangle = \left|r_i\right\rangle \quad \Rightarrow \quad H_D\left|r_i\right\rangle = \omega_{Di}\left|r_i\right\rangle \tag{5-9}$$

Therefore, according to the Composition Postulate CP, at time $t_0 = 0$ the state of the composite system $\mathcal{S} \cup \mathcal{D}$ will be

$$\begin{aligned}\left|\psi_I(t_0 = 0)\right\rangle &= \left|\psi_S(t_0 = 0)\right\rangle \otimes \left|\psi_D\right\rangle = \\ &= \sum_i c_i \left|a_i\right\rangle \otimes \left|r_0\right\rangle \in \mathcal{O} = \mathcal{O}_S \otimes \mathcal{O}_D\end{aligned} \tag{5-10}$$

Since during Stage I there is no interaction between $\mathcal{S}$ and $\mathcal{D}$, then $H_{\text{int}} = 0$ and the total Hamiltonian of $\mathcal{S} \cup \mathcal{D}$ is

$$H = H_S \otimes I_D + I_S \otimes H_D \in \mathcal{O} \tag{5-11}$$

Stage II: In this second, interaction stage, the systems $\mathcal{S}$ and $\mathcal{D}$ interact through an interaction Hamiltonian H_{int}. This means that the composite system $\mathcal{S} \cup \mathcal{D}$ becomes the system $\mathcal{S}_{\text{II}} : (\mathcal{O}, H_{\text{II}})$, whose Hamiltonian reads

$$H_{\text{II}} = H_S \otimes I_D + I_S \otimes H_D + H_{\text{int}} = H + H_{\text{int}} \in \mathcal{O} \tag{5-12}$$

In turn, the state $\left|\psi_{\text{I}}(t_0 = 0)\right\rangle$ of $\mathcal{S} \cup \mathcal{D}$ in Stage I turns out to be the initial state $\left|\psi_{\text{II}}(t_0 = 0)\right\rangle$ of $\mathcal{S}_{\text{II}}$ in Stage II, which evolves to a state $\left|\psi_{\text{II}}(t_1)\right\rangle$ after a $\Delta t = t_1$:

$$\left|\psi_{\text{II}}(t_1)\right\rangle = e^{-iH_{\text{II}}t_1/\hbar}\left|\psi_{\text{II}}(t_0 = 0)\right\rangle = e^{-iH_{\text{II}}t_1/\hbar}\left|\psi_{\text{I}}(t_0 = 0)\right\rangle \tag{5-13}$$

It can be proved that, if the interaction Hamiltonian H_{int} is

$$H_{\text{int}} = -\frac{\lambda\hbar}{t_1}\left(A \otimes P^R\right) \tag{5-14}$$

where λ is a constant and P^R is the observable conjugate to R, $\left[R, P^R\right] = i\hbar$, then the final state of $\mathcal{S}_{\text{II}}$ in Stage II is (see Mittelstaedt 1998)

$$\left|\psi_{\text{II}}(t_1)\right\rangle = \sum_i c_i \left|a_i\right\rangle \otimes \left|r_i\right\rangle \tag{5-15}$$

Stage III: At time $t = t_1$ the interaction ends: the system $\mathcal{S}_{\text{II}}$ becomes the original composite system $\mathcal{S} \cup \mathcal{D}$, whose Hamiltonian is again $H = H_S \otimes I_D + I_S \otimes H_D \in \mathcal{O}$. In turn, the state $\left|\psi_{\text{II}}(t_1)\right\rangle$ of $\mathcal{S}_{\text{II}}$ in Stage II becomes the initial state $\left|\psi_{\text{III}}(t_1)\right\rangle$ of $\mathcal{S} \cup \mathcal{D}$ in Stage III. Since in this stage $\mathcal{S}$ and $\mathcal{D}$ are again elemental quantum systems, we can apply the Actualization Rule to each one of them:

c) In the device $\mathcal{D}$, the initial state of the third stage, $\rho_D = Tr_S\left|\psi_{\text{III}}(t_1)\right\rangle\left\langle\psi_{\text{III}}(t_1)\right|$, evolves unitarily under the action of H_D. Nevertheless, the preferred context is time-invariant: since H_D is the non-degenerate Hamiltonian of a macroscopic system and $\left[H_D, R\right] = 0$, both H_D and R are actual-valued.

d) In the system $\mathcal{S}$, the initial state of the third stage, $\rho_S(t_1) = Tr_D\left|\psi_{\text{III}}(t_1)\right\rangle\left\langle\psi_{\text{III}}(t_1)\right|$, evolves unitarily under the action of H_S. However, two cases have to be distinguished. If $\left[H_S, A\right] = 0$, then both H_S and A may have actual values. But if $\left[H_S, A\right] \neq 0$, the observable A is not actual-valued.

The fact that in certain situations the observable A of the system $\mathcal{S}$ may have no actual value turns out to be non-problematic when we recall that the goal of a quantum system is not to "discover" the value of the observable A, but to know the coefficients c_i of the systems's state $\left|\psi_S\right\rangle$. In fact, the coefficients can be obtained by registering the frequencies of detection of each eigenvalue of R, since

$$\rho_D(t_1) = Tr_S\left|\psi_{\text{III}}(t_1)\right\rangle\left\langle\psi_{\text{III}}(t_1)\right| = \sum_{ij} c_i c_j^* \left|r_i\right\rangle\left\langle r_j\right| \tag{5-16}$$

and the probability corresponding to the eigenvalue r_i can be computed by means of the Born rule:

$$Pr\left(r_i, \rho_D(t_1)\right) = \left\langle r_i\right|\rho_D(t_1)\left|r_i\right\rangle = \left|c_i\right|^2 \tag{5-17}$$

Summing up, according to our MHI, no matter whether the system's observable acquires an actual value or not, the device's pointer is always actual-valued, and the frequencies of those actual values provide us the correct coefficients of the system's state. When the initial state of $\mathcal{S}$ is not pure but mixed, $\rho_S = \sum_{ij}\rho_{ij}\left|a_i\right\rangle\left\langle a_j\right|$, this procedure will supply only the diagonal coefficients ρ_{ii}; if we want to know the remaining coefficients ρ_{ij} with $i \neq j$, we have to perform further measurements with different experimental arrangements (see Ballentine 1998).

5.2. NON-IDEAL MEASUREMENT

Having rejected the collapse hypothesis to solve the measurement problem, many non-collapse interpretations were specifically designed to supply an alternative answer to the problem. Usually those attempts work with no difficulties in the case of ideal measurements, but run into troubles when

faced to non-ideal measurements. This fact cannot be ignored, since ideal measurement is a situation that can never be achieved in practice: the interaction between the measured system and the measuring device never introduces an absolutely perfect correlation. In spite of this, successful measurements are commonly performed in real experiments. So, we are committed to show that our MHI is able to account for quantum measurements even in non-ideal situations.

Two kinds of non-ideal measurements are usually distinguished in the literature:

- **Imperfect measurement (first kind):**

$$\sum_i c_i |a_i\rangle \otimes |r_0\rangle \;\; \rightarrow \;\; \sum_{ij} d_{ij} |a_i\rangle \otimes |r_j\rangle \tag{5-18}$$

where, in general, $d_{ij} \neq 0$ with $i \neq j$

- **Disturbing measurement (second kind):**

$$\sum_i c_i |a_i\rangle \otimes |r_0\rangle \;\; \rightarrow \;\; \sum_i c_i |a_i^d\rangle \otimes |r_i\rangle \tag{5-19}$$

where, in general, $\langle a_i^d | a_j^d \rangle \neq \delta_{ij}$

However, the disturbing measurement can also be expressed as an imperfect measurement by a change of basis:

$$\sum_i c_i |a_i^d\rangle \otimes |r_i\rangle = \sum_{ij} d_{ij} |a_i\rangle \otimes |r_j\rangle \tag{5-20}$$

In certain modal interpretations, the rule of property-ascription, when applied to non-ideal measurements, leads to results that disagree with those obtained in the orthodox collapse interpretation (see Albert and Loewer 1990, 1993). If the properties ascribed by a modal interpretation are different from those ascribed by the collapse interpretation, the question is how different they are. In the case of an imperfect measurement, it can be expected that the $d_{ij} \neq 0$, with $i \neq j$, be small; then, the difference might be also small. But in the case of a disturbing measurement, the $d_{ij} \neq 0$, with $i \neq j$, need not be small and, as a consequence, the disagreement between the properties ascribed

by the modal interpretation and those ascribed by collapse might be unacceptable (see a full discussion in Bacciagaluppi and Hemmo 1996). This fact has been considered by Harvey Brown as a "silver bullet" for killing the modal interpretations (cited in Bacciagaluppi and Hemmo 1996).

We shall not distinguish between the two kinds of non-ideal measurements because the result of the application of our Actualization Rule does not depend on the values of the off-diagonal terms d_{ij}. As we shall see, according to the MHI, the observable R that plays the role of the device's pointer acquires an actual value in any case.

Stages I to III: In a non-ideal measurement, Stage I is characterized in the same way as in the ideal case. The difference begins at Stage II, where the correlation introduced by the interaction Hamiltonian H_{int} is not perfect. Therefore, the final state $\left|\psi_{\text{II}}(t_1)\right\rangle$ of Stage II, which is the initial state $\left|\psi_{\text{III}}(t_1)\right\rangle$ of Stage III, reads

$$\left|\psi_{\text{III}}(t_1)\right\rangle = \left|\psi_{\text{II}}(t_1)\right\rangle = \sum_{ij} d_{ij} \left|a_i\right\rangle \otimes \left|r_j\right\rangle \tag{5-21}$$

As discussed in the case of the ideal measurement, we are not interested in the actual-valued observables of $\mathcal{S}$; so, we shall analyze the result of the process in the device $\mathcal{D}$, which begins Stage III in an initial state

$$\begin{aligned} \rho_D(t_1) &= Tr_S \left|\psi_{\text{III}}(t_1)\right\rangle \left\langle\psi_{\text{III}}(t_1)\right| = \\ &= \sum_n \left\langle a_n \left|\psi_{\text{III}}(t_1)\right\rangle \left\langle\psi_{\text{III}}(t_1)\right| a_n \right\rangle = \sum_{ij} \rho_{Dij} \left|r_i\right\rangle \left\langle r_j\right| \end{aligned} \tag{5-22}$$

where

$$\rho_{Dij} = \sum_n d_{ni} d_{nj}^* \tag{5-23}$$

Although $\mathcal{D}$ evolves unitarily under the action of H_D, the preferred context is time-invariant since it is defined by the eigenbasis of H_D. In turn, since H_D commutes with R, both H_D and R are actual-valued.

Also in this case, the coefficients can be obtained by registering the frequencies of detection of each eigenvalue of R:

$$Pr\left(r_i, \rho_D(t_1)\right) = \left\langle r_i \left| \rho_D(t_1) \right| r_i \right\rangle = \rho_{Dii} = \sum_n \left|d_{ni}\right|^2 = \left|d_{ii}\right|^2 + \sum_{n \neq i} \left|d_{ni}\right|^2 \quad (5\text{-}24)$$

As we can see, if the coefficients d_{ni}, with $n \neq i$, of the off-diagonal terms of the initial state in Stage III are zero, we are in the ideal measurement case, where $\rho_{Dii} = \left|d_{ii}\right|^2 = \left|c_i\right|^2$. If the coefficients d_{ni}, with $n \neq i$, are not zero, we are in the non-ideal measurement case. However, in this case two situations have to be distinguished:

- If the d_{ni}, with $n \neq i$, are small in the sense that $\sum_{n \neq i} \left|d_{ni}\right|^2 \ll \left|d_{ii}\right|^2$, then $\rho_{Dii} \simeq \left|d_{ii}\right|^2 \simeq \left|c_i\right|^2$ (see eq. (5-24)). This means that, by the repetition of detection, the coefficients $\left|c_i\right|^2$ can be approximately obtained, and the measurement will be *reliable.*
- If the d_{ni}, with $n \neq i$, are not small, then $\rho_{Dii} \simeq \left|d_{ii}\right|^2$ does not hold. Therefore, the result obtained by means of the measurement will be *non-reliable.*

Summing up, our MHI can account for the fact that perfect correlation is not a necessary condition for "good" measurements: if the reliability condition of small cross-terms is satisfied, the coefficients of the system's state at the beginning of the process can be approximately computed even when the correlation is not perfect. Nevertheless, both in the reliable and in the non-reliable case, in each measurement an actual reading of the pointer is obtained.

5.3. THE STERN-GERLACH EXPERIMENT

Since the Stern-Gerlach experiment is the paradigm of quantum measurement, it is worth while to see how all the elements of our general account of measurement can be found in this case.

The experiment is usually described as follows. A neutral free particle with spin, with constant velocity in the y-direction, passes between the poles of a magnet that produces an inhomogeneous magnetic field $\boldsymbol{B}$, with components $B_x = B_y = 0$ and $B_z = zB'$, where B' is the field gradient. The particle is described in the plane zy, and in a frame of reference moving uniformly in the y-direction, where $P_y = 0$. The gradient of the magnetic

field produces a force that deflects the particle in the z-direction: the deflection depends on the component of spin in that direction.

As we have seen in Section 4.2, the free particle with spin is a composite system $\mathcal{S}_s \cup \mathcal{S}_f$. In this measurement situation:

- the spin subsystem $\mathcal{S}_s$, represented in $\mathcal{H}_s$ and with Hamiltonian $H_s = k S^2$, is the system under measurement $\mathcal{S}$.
- the free particle without spin $\mathcal{S}_f$, represented in $\mathcal{H}_f$ and with Hamiltonian $H_f = P_z^2 / 2m$, has to be a part of a measuring device $\mathcal{D}$ such that $[H_D, P_z] = 0$: this guarantees that the eigenvectors of P_z are stationary and, then, P_z can play the role of the pointer.

On this basis, at Stage I we find that:

- The observable A is the spin in z-direction, $S_z \in \mathcal{O}_S = \mathcal{H}_S \otimes \mathcal{H}_S$:

$$S_z|\uparrow\rangle = s_\uparrow|\uparrow\rangle \qquad S_z|\downarrow\rangle = s_\downarrow|\downarrow\rangle \tag{5-25}$$

  where $s_\uparrow = -s_\downarrow = (1/2)\hbar$.

- The momentum in z-direction plays the role of the pointer, $P_z \in \mathcal{O}_D = \mathcal{H}_D \otimes \mathcal{H}_D$:

$$P_z|+\rangle = p_+|+\rangle \qquad P_z|-\rangle = p_-|-\rangle \qquad P_z|0\rangle = p_0|0\rangle \tag{5-26}$$

  where $\{|+\rangle, |-\rangle, |0\rangle\}$ is a basis of $\mathcal{H}_D$.

- The states of $\mathcal{S}$ and $\mathcal{D}$ are, respectively, $|\psi_S\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$ and $\psi_D = |0\rangle$. Then,

$$|\psi_I(t_0 = 0)\rangle = c_1|\uparrow\rangle \otimes |0\rangle + c_2|\downarrow\rangle \otimes |0\rangle \tag{5-27}$$

- As we have said, the Hamiltonian of $\mathcal{S}$ is $H_S = k S^2$, and the Hamiltonian of $\mathcal{D}$ is such that $[H_D, P_z] = 0$. Therefore,

$$H_D|+\rangle = \omega_+|+\rangle \qquad H_D|-\rangle = \omega_-|-\rangle \qquad H_D|0\rangle = \omega_0|0\rangle \tag{5-28}$$

Ideal measurement: At Stage II, the total Hamiltonian $H_{\text{II}} = H_S + H_D + H_{\text{int}}$ introduces a perfect correlation. Then, the initial state of $\mathcal{S} \cup \mathcal{D}$ in Stage III is

$$\left|\psi_{\text{III}}(t_1)\right\rangle = c_1 \left|\uparrow\right\rangle \otimes \left|+\right\rangle + c_2 \left|\downarrow\right\rangle \otimes \left|-\right\rangle \tag{5-29}$$

The initial state of the subsystem $\mathcal{D}$ then reads

$$\rho_D(t_1) = Tr_S \left|\psi_{\text{III}}(t_1)\right\rangle \left\langle\psi_{\text{III}}(t_1)\right| = \left|c_1\right|^2 |+\rangle\langle+| + \left|c_2\right|^2 |-\rangle\langle-| \tag{5-30}$$

Since $\left[H_D, P_z\right] = 0$, both the Hamiltonian H_D and the momentum P_z in z-direction are actual-valued. The probabilities corresponding to the possible readings can be computed as

$$Pr\left(+, \rho_D(t_1)\right) = \left\langle + \left| \rho_D(t_1) \right| + \right\rangle = \left|c_1\right|^2 \tag{5-31}$$

$$Pr\left(-, \rho_D(t_1)\right) = \left\langle - \left| \rho_D(t_1) \right| - \right\rangle = \left|c_2\right|^2 \tag{5-32}$$

$$Pr\left(0, \rho_D(t_1)\right) = \left\langle 0 \left| \rho_D(t_1) \right| 0 \right\rangle = 0 \tag{5-33}$$

As expected, these measures are time-invariant: they do not depend on the time when the reading of the pointer is performed, that is, on the precise position where the detectors are placed in Stage III. If the probabilities depended on the instantaneous state of the system, the result of the measurement would be extremely sensitive to the precise location of the detectors: any imperceptible perturbation would substantially modify the frequencies so obtained, making the measurement of the $\left|c_i\right|^2$ physically unrealizable.

Non-ideal measurement: In this case, H_{II} does not introduce a perfect correlation. The initial state of $\mathcal{S} \cup \mathcal{D}$ in Stage III is, then,

$$\begin{aligned} \left|\psi_{\text{III}}\right\rangle = {} & d_{11} \left|\uparrow\right\rangle \otimes \left|+\right\rangle + d_{12} \left|\uparrow\right\rangle \otimes \left|-\right\rangle + \\ & + d_{21} \left|\downarrow\right\rangle \otimes \left|+\right\rangle + d_{22} \left|\downarrow\right\rangle \otimes \left|-\right\rangle \end{aligned} \tag{5-34}$$

The initial state of the subsystem $\mathcal{D}$ reads

$$\begin{aligned}\rho_D(t_1) = \rho_{D11}\,|+\rangle\langle+| + \rho_{D12}\,|+\rangle\langle-| + \\ + \rho_{D21}\,|-\rangle\langle+| + \rho_{D22}\,|-\rangle\langle-|\end{aligned} \tag{5-35}$$

where

$$\rho_{Dij} = \sum_{n=1}^{2} d_{ni} d_{nj}^{*} \tag{5-36}$$

In other words,

$$\rho_D(t_1) = \begin{pmatrix} |d_{11}|^2 + |d_{21}|^2 & d_{11}d_{12}^{*} + d_{21}d_{22}^{*} \\ d_{12}d_{11}^{*} + d_{22}d_{21}^{*} & |d_{12}|^2 + |d_{22}|^2 \end{pmatrix} \tag{5-37}$$

Again, since $[H_D, P_z] = 0$, both the Hamiltonian H_D and the momentum P_z in z-direction are actual-valued. But now the probabilities corresponding to the possible readings are

$$Pr(+, \rho_D(t_1)) = \langle + | \rho_D(t_1) | + \rangle = |d_{11}|^2 + |d_{21}|^2 \tag{5-38}$$

$$Pr(+, \rho_D(t_1)) = \langle - | \rho_D(t_1) | - \rangle = |d_{22}|^2 + |d_{12}|^2 \tag{5-39}$$

$$Pr(+, \rho_D(t_1)) = \langle 0 | \rho_D(t_1) | 0 \rangle = 0 \tag{5-40}$$

In this non-ideal case, the measurement will be reliable if $|d_{21}|^2 \ll |d_{11}|^2$ and $|d_{12}|^2 \ll |d_{22}|^2$; if not, the measurement will not supply the necessary information for the reconstruction of the original state of the measured system. Nevertheless, the observable P_z acquires an actual value in any case, and this is the prediction that can be directly tested in each single detection.

This analysis of the Stern-Gerlach experiment allows us to point out a feature of the quantum measurement that cannot be noticed in the merely formal treatments of the process. In fact, in the von Neumann model, the

observable A of the system $\mathcal{S}$ under measurement is considered in formal terms and deprived of its physical content. Then, the interaction between $\mathcal{S}$ and the measuring device $\mathcal{D}$ is endowed with the only role of introducing the correlation between A and the pointer R. However, the varied physical situations described in Chapter 4 show that we have no empirical access to the observables that are generators of the symmetries of the system's Hamiltonian; in the context of measurement, A may be one of those observables. This is precisely the case in the Stern-Gerlach experiment, where S_z is a generator of the space-rotation symmetry of $H_s = k S^2$. It is the interaction with the magnetic field B_z what breaks the isotropy of space by privileging the z-direction and, as a consequence, breaks the space-rotation symmetry of H_s (see Section 4.2). This physical account of the measurement shows that, when the observable A is a generator of a symmetry of the Hamiltonian H_S of $\mathcal{S}$, the interaction with the device $\mathcal{D}$ has to break that symmetry and, at the same time, has to establish the correlation between A and R. Therefore, from a physical viewpoint, measurement can be conceived as a process that breaks the symmetries of the system to be measured and, in this way, allows us to reconstruct its state in terms of an otherwise empirically inaccessible symmetry-generator observable. The idea is that the formal von Neumann model of quantum measurement can be complemented by a physical model in terms of which measurement is a symmetry-breaking process that renders a symmetry generator of the system's Hamiltonian empirically accessible.

5.4. INFINITE TAILS

An argument that stresses the difficulties introduced by non-ideal measurements is that posed by Elby (1993) in the context of the Stern-Gerlach experiment. This argument points to the fact that the wavefunctions in z-variable typically have infinite "tails" that introduce non-zero cross-terms; therefore, the "tail" of the wavefunction of the "down" beam may produce detection in the upper detector, prepared to detect p_+, and vice versa.

Let us consider this new argument in detail by supposing that the imperfection is due to a non-perfect collimation of the incoming beam. In this case, with the magnetic field still turned off, we would obtain a diffuse spot instead of a definite point on the screen. Therefore, the perfect ready-to-

measure state $|r_0\rangle = |0\rangle$ has to be replaced with a narrow Gaussian $|\varphi_0(z)\rangle$. As a consequence, the measurement process turns out to be expressed as

$$\begin{aligned} |\psi_{\mathrm{I}}\rangle &= \left(c_1|\uparrow\rangle + c_2|\downarrow\rangle\right)\otimes|\varphi_0(z)\rangle \rightarrow \\ &\rightarrow |\psi_{\mathrm{III}}\rangle = c_1|\uparrow\rangle\otimes|\varphi_+(z)\rangle + c_2|\downarrow\rangle\otimes|\varphi_-(z)\rangle \end{aligned} \tag{5-41}$$

where now $|\varphi_+(z)\rangle$ and $|\varphi_-(z)\rangle$ are Gaussians that do not need to be as narrow as the initial one. Let us call the widths of the upper and the lower detectors Δz_+ and Δz_- respectively. Thus, the long tail of the Gaussian $|\varphi_+(z)\rangle$ arrives to Δz_- and the long tail of the Gaussian $|\varphi_-(z)\rangle$ arrives to Δz_-. We can compute the probabilities corresponding to the four possible cases:

$$p(\uparrow,+) = \left|\left(\langle\uparrow|\otimes\langle\varphi_+(z)|\right)|\psi_{\mathrm{III}}\rangle\right|^2 = |c_1|^2\int_{\Delta z_+}\left\|\varphi_+(z)\right\rangle\Big|^2 dz = |c_{11}|^2 \tag{5-42}$$

$$p(\uparrow,-) = \left|\left(\langle\uparrow|\otimes\langle\varphi_-(z)|\right)|\psi_{\mathrm{III}}\rangle\right|^2 = |c_1|^2\int_{\Delta z_-}\left|\langle\varphi_-(z)|\varphi_+(z)\rangle\right|^2 dz = |c_{12}|^2 \tag{5-43}$$

$$p(\downarrow,+) = \left|\left(\langle\downarrow|\otimes\langle\varphi_+(z)|\right)|\psi_{\mathrm{III}}\rangle\right|^2 = |c_2|^2\int_{\Delta z_+}\left|\langle\varphi_+(z)|\varphi_-(z)\rangle\right|^2 dz = |c_{21}|^2 \tag{5-44}$$

$$p(\downarrow,-) = \left|\left(\langle\downarrow|\otimes\langle\varphi_-(z)|\right)|\psi_{\mathrm{III}}\rangle\right|^2 = |c_2|^2\int_{\Delta z_-}\left\|\varphi_-(z)\right\rangle\Big|^2 dz = |c_{22}|^2 \tag{5-45}$$

where

$$Pr(+,\rho_D(t_1)) = p(\uparrow,+) + p(\downarrow,+) = |c_{11}|^2 + |c_{21}|^2 \tag{5-46}$$

$$Pr(+,\rho_D(t_1)) = p(\downarrow,-) + p(\uparrow,-) = |c_{22}|^2 + |c_{12}|^2 \tag{5-47}$$

According to Elby's argument, these cases can be read as follows:

- $|c_{11}|^2$ is the probability that $|\uparrow\rangle$ be detected by Δz_+
- $|c_{12}|^2$ is the probability that $|\uparrow\rangle$ be detected by Δz_- (tail)
- $|c_{21}|^2$ is the probability that $|\downarrow\rangle$ be detected by Δz_+ (tail)
- $|c_{22}|^2$ is the probability that $|\downarrow\rangle$ be detected by Δz_-

Our MHI shows that, if the reliability condition $|c_{21}|^2 \ll |c_{11}|^2$ and $|c_{12}|^2 \ll |c_{22}|^2$ holds, then the collimation, even if not perfect, is good enough for measurement, since $|c_{11}|^2 \simeq |c_1|^2$ and $|c_{22}|^2 \simeq |c_2|^2$. If the original Gaussian is not very narrow or the screen is placed too far from the magnet, the measurement will be non-reliable since the c_{ij}, with $i \neq j$, are not small enough. Nevertheless, according to the Actualization Rule, since the preferred context is defined by the eigenbasis of H_D and the pointer commutes with H_D, we obtain an actual reading of the pointer, that is, an actual detection in Δz_+ or Δz_-.

Chapter 6

INTERPRETATION AND DECOHERENCE

As pointed out, each modal interpretation proposes a specific interpretative rule of actual-value ascription, in general with the aim of offering and adequate answer to the quantum measurement problem. Some of them work very well in the account of ideal measurements, but face severe problems in the non-ideal case. It is at this point that environment-induced decoherence (EID) entered the discussion: some authors suggested that, since measuring apparatuses are never isolated from their environments, decoherence provides an answer to the non-ideal-measurement challenges (see Healey 1989, 1995; Dieks 1994a, 1994b; Bacciagaluppi and Hemmo 1996; Monton 1999).

Since immune to the non-ideal-measurement challenges, the MHI has no need of decoherence for giving an adequate account of quantum measurement. Nevertheless, to the extent that the preferred context is defined by the Hamiltonian of the system –conceived as a closed system with no external interaction–, the MHI seems to be incompatible with the EID approach, which relies on the interaction between the measurement apparatus –an open system– and its environment. Although the theory of decoherence does not supply an interpretation of quantum mechanics, given its impressive success nowadays no interpretation can ignore its results. Therefore, the incompatibility between MHI and EID would count against our interpretation. In this chapter we shall argue that the conflict is merely apparent: in the measurement situation, the preferred context defined by the MHI agrees with the pointer basis selected by EID.

6.1. THE ENVIRONMENT-INDUCED POINTER BASIS

In his first papers on decoherence, Zurek (1981, 1982) studied physical models where the reduced density matrix ends up being diagonal in the eigenvectors of an observable R, which commutes with the Hamiltonian H_{ME}^{int} describing the apparatus-environment interaction. According to Zurek, this property is what makes R to be the pointer observable: since R is a constant of motion of H_{ME}^{int}, when the apparatus is in one of its eigenstates, the interaction with the environment will leave it unperturbed: "*The form of the interaction Hamiltonian between the apparatus and its environment is sufficient to determine which observable of the measured quantum system can be considered «recorded» by the apparatus. The basis that contains that record –the pointer basis of the apparatus– consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian*" (Zurek 1981, p. 1516). Since those first works, the condition $\left[R, H_{ME}^{\text{int}}\right] = 0$ has usually been considered as the definition of the pointer basis or of the pointer R of the apparatus. For instance, Elby (1994, p. 363) explains: "*Let P' denote an arbitrary apparatus observable that doesn't commute with the pointer reading P. Using 'toy' examples, along with general considerations, Zurek argues that H_{ME}^{int} commutes with P, but does not commute with any P'. In rough terms, the interaction between the apparatus and its environment picks out the pointer-reading basis*". More recently, Schlosshauer (2004, pp. 1278-1279) claims: "*One can then find a sufficient criterion for dynamically stable pointer states that preserve the system-apparatus correlations in spite of the interaction of the apparatus with the environment by requiring all pointer state projection operators $P_n = \left|p_n\right\rangle\left\langle p_n\right|$ to commute with the apparatus-environment Hamiltonian H_{ME}^{int}.*"

In the 90's, Zurek stressed that the original definition of the pointer basis was a simplification: when the system's dynamics is relevant, the einselection of the preferred basis is more complicated. Zurek introduced the "*predictability sieve*" criterion (Zurek 1993, Zurek, Habib and Paz 1993) as a systematic strategy to identify the preferred basis in generic situations. The criterion relies on the fact that the preferred states are, by definition, those less affected by the interaction with the environment. On the basis of the application of this criterion, three different regimes for the selection of the

preferred basis can be distinguished (Paz and Zurek 1999, 2002; see also Zurek 2003):

- The first regime is the quantum measurement situation, where the self-Hamiltonian of the system can be neglected and the evolution is completely dominated by the interaction Hamiltonian. In such a case, the preferred states are directly the eigenstates of the interaction Hamiltonian (Zurek 1981).
- The second regime is the more realistic and complex situation, where neither the self-Hamiltonian of the system nor the interaction with the environment are clearly dominant, but both induce non-trivial evolution. In this case, the preferred basis arises from the interplay between self-evolution and interaction; quantum Brownian motion belongs to this case (Paz 1994).
- The third regime corresponds to the situation where the dynamics is dominated by the system's self-Hamiltonian. In this case, the preferred states are simply the eigenstates of this self-Hamiltonian (Paz and Zurek 1999).

6.2. THE MODAL-HAMILTONIAN PREFERRED CONTEXT

On the basis of the description of the measurement process given in Chapter 5, it is clear that, according to HMI, two conditions define a quantum measurement:

(a) During a period Δt, the measured system $\mathcal{S}$ and the measuring device $\mathcal{D}$ must interact through an interaction Hamiltonian $H_{SD}^{\text{int}} \neq 0$ intended to introduce a correlation between the observable A of $\mathcal{S}$ and the pointer R of $\mathcal{D}$. The requirement of perfect correlation is not included as a defining condition of measurement, because the Actualization Rule explains the actual reading of the pointer R even in non-ideal measurements, that is, when the correlation is not perfect.

(b) The measuring device $\mathcal{D}$ has to be constructed in such a way that its pointer R (i) has macroscopically distinguishable eigenvalues, and (ii) commutes with the Hamiltonian H_D and has, at least, the same degeneracy as H_D.

The first step towards dissolving the seeming conflict between MHI and EID is to understand that, in the account given by MHI, the measuring device $\mathcal{D}$ is not the macroscopic apparatus $\mathcal{A}$ designed by the experimentalist for measurement (eventually surrounded by a "bath" $\mathcal{B}$ of particles in interaction with it), but the entire quantum system that interacts with the system $\mathcal{S}$ in the second stage and remains closed in the third stage: it is this system what must have a pointer R commuting with its Hamiltonian H_D. On this basis, we can now analyze the elements that participate in the process as described in the framework of the MHI:

The closed system $\mathcal{D}$ –e.g., the apparatus $\mathcal{A}$ *plus* the bath of particles $\mathcal{B}$– is certainly a macroscopic system, whose Hamiltonian is the result of the interaction among a huge number of degrees of freedom. Since, in general, symmetries are broken by interactions, the symmetry of a Hamiltonian decreases with the complexity of the system. Then, a macroscopic system having a Hamiltonian with symmetries is a highly exceptional situation: in the generic case, the energy is the only constant of motion of the macroscopic system. As a consequence, in realistic measurement situations, H_D is non-degenerate (see eq. (5-8),

$$H_D\left|\omega_{Di}\right\rangle = \omega_{Di}\left|\omega_{Di}\right\rangle \quad \text{where } \left\{\left|\omega_{Di}\right\rangle\right\} \text{ is a basis of } \mathcal{H}_D \tag{6-1}$$

This means that, when $\left[R,H_D\right]=0$, we can guarantee that R has, at least, the same degeneracies as H_D because H_D is non-degenerate.

The pointer R cannot have such a huge number of different eigenvalues as H_D, because the experimental physicist must be able to discriminate among them (for instance, in the Stern-Gerlach experiment the pointer has three eigenvalues). This means that R is a "collective" observable of $\mathcal{D}$ (see Omnés 1994, 1999), that is, a highly degenerate observable that does not "see" the vast majority of the degrees of freedom of $\mathcal{D}$:

$$R = \sum_n r_n R_n \tag{6-2}$$

where the set $\left\{R_n\right\}$ of the eigenprojectors of R spans the Hilbert space $\mathcal{H}_D$ of $\mathcal{D}$. In other words, the eigenprojectors of R introduce a sort of "coarse-graining" onto the Hilbert space $\mathcal{H}_D$. Therefore, if the Hamiltonian H_D is

non-degenerate, the condition $\left[R,H_D\right]=0$ (see eq. (5-9)) implies that R can be expressed in terms of the energy eigenbasis $\left\{\left|\omega_{Di}\right\rangle\right\}$ as

$$R=\sum_n r_n R_n=\sum_n r_n \sum_{i_n}\left|\omega_{Di_n}\right\rangle\left\langle\omega_{Di_n}\right| \tag{6-3}$$

This expression shows that, since $r_n \neq r_{n'}$, R has more degeneracies than H_D.

The requirement $\left[R,H_D\right]=0$, far from being an *ad hoc* condition necessary to apply the modal-Hamiltonian Actualization Rule, has a clear physical meaning: it is essential to preserve the stationary behavior of R during the third stage of the measurement process, in order to make the reading of R possible. If this requirement did not hold because of the uncontrollable interaction among the microscopic degrees of freedom of the macroscopic apparatus or between the macroscopic apparatus and an external "bath", the reading of R would constantly change and measurement would be impossible. Therefore, the complete experimental arrangement has to be designed in such a way that the uncontrollable degrees of freedom of $\mathcal{D}$ do not affect significantly the stationarity of the pointer. This goal may be achieved by many different technological means; but, in any case, measurement has to be a controlled situation where the reading of a stable pointer can be obtained

6.3. THE ENVIRONMENT-INDUCED POINTER BASIS FROM A CLOSED-SYSTEM PERSPECTIVE

In the context of EID, during the third stage the measuring apparatus $\mathcal{M}$ does no longer interact with the measured system $\mathcal{S}$ but interacts with the environment $\mathcal{E}$. If, in the context of MHI, we use $\mathcal{D}=\mathcal{M}+\mathcal{E}$ to call the whole system that interacts with $\mathcal{S}$ in the second stage but remains closed during the third stage, the question is how to identify the open interacting parts of $\mathcal{D}$ to be conceived as the measuring apparatus $\mathcal{M}$ and the environment $\mathcal{E}$. This is a legitimate question because, as we stressed from the very beginning, a whole closed system may be partitioned in many different ways,

none of them more "essential" than the others (Harshman and Wickramasekara 2007).

A natural assumption is to consider the macroscopic, material apparatus $\mathcal{A}$ built for measurement as "the measuring apparatus" $\mathcal{M}$, and the bath $\mathcal{B}$ of the particles scattering off $\mathcal{A}$ as "the environment" $\mathcal{E}$; then, $\mathcal{D} = \mathcal{A} + \mathcal{B}$ is the closed system resulting from the interaction between $\mathcal{A}$ and $\mathcal{B}$. From this position, it is supposed that $\mathcal{A}$ is the open system that decoheres: the reduced density operator $\rho_r^A(t)$ of $\mathcal{A}$ should converge to a final time-independent ρ_r^A, diagonal in the pointer basis of $\mathcal{A}$, that is, of its Hilbert space $\mathcal{H}_A$, and the pointer R should define such a basis. However, although apparently "natural", this is not the best choice for the split of $\mathcal{D}$, since it does not take into account the environment *internal* to the device $\mathcal{A}$. In fact, being a macroscopic body, $\mathcal{A}$ has a huge number of degrees of freedom, which have to be "coarse-grained" by R if it is to play the role of the pointer. In other words, since the pointer R must have a small number of different eigenvalues to allow the observer to discriminate among them, R is a highly degenerate observable on the Hilbert space $\mathcal{H}_A$ of the open macroscopic apparatus $\mathcal{A}$ and, as a consequence, it does not define a *basis* of $\mathcal{H}_A$.

Since a closed quantum system can be partitioned in many, equally legitimate manners, $\mathcal{D}$ can be split in a theoretically better founded way in the measurement case. Let us recall that the pointer R is the observable whose eigenvectors became correlated with the eigenvectors of an observable of the measured system during the second stage of the process, and that the interaction in that stage was deliberately designed to introduce such a correlation. So, if we want that during the third stage R really defines a basis, the open "measuring apparatus" $\mathcal{M}$ must be the part of $\mathcal{D}$ corresponding to the Hilbert space $\mathcal{H}_M$ where the pointer is non-degenerate. If we call R_M the pointer belonging to $\mathcal{H}_M \otimes \mathcal{H}_M$, it reads

$$R_M = \sum_n r_n \left| r_n \right\rangle \left\langle r_n \right| \tag{6-4}$$

where $\left\{ \left| r_n \right\rangle \right\}$ is a basis of $\mathcal{H}_M$. Then, the relevant partition is $\mathcal{H}_D = \mathcal{H}_M \otimes \mathcal{H}_E$, where $\mathcal{H}_E$ is the Hilbert space of the "environment" $\mathcal{E}$, with basis $\left\{ \left| e_m \right\rangle \right\}$. Then, the pointer acting on $\mathcal{H}_D$ can be expressed as a highly degenerate observable

$$R = R_M \otimes I_E = \left(\sum_n r_n \left| r_n \right\rangle \left\langle r_n \right| \right) \otimes \left(\sum_m \left| e_m \right\rangle \left\langle e_m \right| \right) = \\ = \sum_n r_n \sum_m \left| r_n \right\rangle \otimes \left| e_m \right\rangle \left\langle r_n \right| \otimes \left\langle e_m \right| = \sum_n r_n R_n \tag{6-5}$$

This agrees with the features of R required by MHI: R introduces a sort of "coarse-graining" onto the Hilbert space $\mathcal{H}_D$ (compare eq. (6-5) with eq. (6-2)). The many degrees of freedom corresponding to the degeneracies of R in $\mathcal{H}_D$ play the role of the "environment" $\mathcal{E}$, composed by the microscopic degrees of freedom of the macroscopic apparatus $\mathcal{A}$ –internal environment– and the degrees of freedom of the bath $\mathcal{B}$ –external environment–.

6.4. COMPATIBILITY BETWEEN INTERPRETATION AND DECOHERENCE

As we have seen, in the first papers on decoherence, the condition $\left[R, H_{ME}^{\text{int}} \right] = 0$ was considered as the definition of the pointer basis. However, this definition involves several assumptions. In fact, the entangled state $\left| \psi_{SME}(t) \right\rangle$ of the whole system evolves according to the Schrödinger equation under the action of the total Hamiltonian $H_{SME} = H_S + H_M + H_E + \\ + H_{SM}^{\text{int}} + H_{SE}^{\text{int}} + H_{ME}^{\text{int}}$. So, first it is considered that the system-environment interaction and the system-apparatus interaction are zero: $H_{SE}^{\text{int}} = 0$ and $H_{SM}^{\text{int}} = 0$. This assumption is reasonable on the basis of the design of the measurement arrangement: after a short time, any interaction with the system ends and the subsystem $\mathcal{M} + \mathcal{E}$ follows its independent dynamical evolution; for this reason, also the self-Hamiltonian H_S of the system can be disregarded. Then, the stability of the pointer strictly requires that:

$$\left[R, H_{ME} \right] = 0 \\ \text{with} \quad H_{ME} = H_M \otimes I_E + I_M \otimes H_E + H_{ME}^{\text{int}} \tag{6-6}$$

If we recall that the pointer R is an observable highly degenerate in the –internal and external– degrees of freedom of the environment (see eq. (6-5)), then condition (6-6) results

$$\left[R, H_{ME}\right] = \left[R_M \otimes I_E, H_M \otimes I_E + I_M \otimes H_E + H_{ME}^{\text{int}}\right] = 0 \qquad (6\text{-}7)$$

But since $\left[R_M \otimes I_E, I_M \otimes H_E\right] = 0$, then the stability requirement for the pointer observable becomes that it commutes with the Hamiltonian $H_M \otimes I_E + H_{ME}^{\text{int}}$, where the self-Hamiltonian of the environment is not involved:

$$\left[R, H_M \otimes I_E + H_{ME}^{\text{int}}\right] = 0 \qquad (6\text{-}8)$$

This argument shows that the condition $\left[R, H_{ME}^{\text{int}}\right] = 0$, introduced in the first papers on decoherence, is a particular case that holds only when the self-Hamiltonian of $\mathcal{M}$ can be disregarded.

It is also clear that the three regimes distinguished by Zurek as the result of the application of the predictability sieve turn out to be the three particular cases of condition (6-8), and can be redescribed in terms of that condition:

- When $H_M \otimes I_E \ll H_{ME}^{\text{int}}$, the self-Hamiltonian of $\mathcal{M}$ can be neglected, and then $[R, H_{ME}^{\text{int}}] = 0$. Therefore, the preferred basis is defined by the interaction Hamiltonian H_{ME}^{int}.
- When $H_M \otimes I_E \simeq H_{ME}^{\text{int}}$, neither the self-Hamiltonian of $\mathcal{M}$ nor the interaction with the environment are clearly dominant. In this case, the preferred basis is defined by condition (6-8).
- When $H_M \otimes I_E \gg H_{ME}^{\text{int}}$, the dynamics is dominated by the self-Hamiltonian of $\mathcal{M}$ and, then, $\left[R, H_M \otimes I_E\right] = \left[R_M \otimes I_E, H_M \otimes I_E\right] = = \left[R_M, H_M\right] = 0$. Therefore, the preferred states are simply the eigenstates of H_M.

As a consequence, the fact (noticed by Schlosshauer 2004, p. 1280; see also Schlosshauer 2007, pp. 84-85) that many systems are typically found in energy eigenstates although the interaction Hamiltonian depends on an observable different than energy, far from being surprising, is the necessary consequence of the requirement of stability for the preferred basis. But the point we want to stress here is that, when the EID pointer basis is considered from this closed-system viewpoint, it agrees with the preferred context as defined by the MHI Actualization Rule: in both cases, the pointer/preferred context is given by the Hamiltonian of the whole closed system. In fact, the three regimes identified

and obtained case by case by Zurek turn out to be particular cases of the MHI characterization of the preferred context: if the preferred states are defined by the eigenstates of the Hamiltonian of the whole system, it is not hard to realize that they will depend on the Hamiltonian's component that dominates the whole evolution.

Moreover, from this perspective the first regime can be justified on general grounds. According to Zurek, the first regime is the quantum measurement situation, where the self-Hamiltonian of the measuring system $\mathcal{M}$ can be neglected and the evolution is completely dominated by the interaction Hamiltonian: this means that $H_M \otimes I_E \ll H_{ME}^{\text{int}}$. If, as explained in the previous chapter, $\mathcal{M}$ is the part of the closed system $\mathcal{D}$ "viewed" by the pointer R and the environment carries over almost all the degrees of freedom of $\mathcal{D}$, it seems reasonable to suppose that, in general, the Hamiltonian corresponding to the interaction with that huge number of degrees of freedom is much greater than the self-Hamiltonian of the "small" part defined by the pointer: the condition $H_M \otimes I_E \ll H_{ME}^{\text{int}}$ leading to the first regime turns out to have a physical justification.

Chapter 7

THE PHILOSOPHICAL IMPLICATIONS OF THE INTERPRETATION

From a realist perspective, to interpret a theory amounts to saying how reality would be if the theory were true. Although, in general, physicists agree in their use of the physical language, it is not a self-evident matter what the relation between physical language and reality is: physical theories do not provide their own interpretations. Therefore, if we want to give an interpretation for quantum mechanics, we have to formulate ontological interpretative postulates that define the ontological reference of each term of the theory. In other words, we have to specify which kind of items in the ontology (objects, properties, facts, etc.) is represented by each physical term (systems, observables, states, etc.). In this way we shall be able to say what *ontological categories* populate the quantum mechanical reality: the task of fixing *the ontological reference of the physical language* is unavoidable if we want to understand the picture of reality supplied by our interpretation. In order to distinguish between the physical language and its ontological reference, we shall use the following terminology: the symbol '$[\bullet]$' denotes the ontological item referred to by the word '$\bullet$' of the physical language.

7.1. PROPERTIES OF ELEMENTAL AND COMPOSITE SYSTEMS

One of the main areas of controversy in contemporary metaphysics is the problem of the nature of individuals or particular objects: is an individual a substratum supporting properties or a mere "bundle" of properties? (for a survey, see Loux 1998). The idea of a substratum acting as a bearer of properties and/or as the principle of individuation has pervaded the history of philosophy. For instance, it is present under different forms in Aristotle's "primary substance", in Locke's doctrine of "substance in general" or in Leibniz's monads. Nevertheless, many philosophers belonging to the empiricist tradition, from Hume to Russell, Ayer and Goodman, have considered the posit of a characterless substratum as a metaphysical abuse. As a consequence, they have adopted some version of the "bundle theory", according to which an individual is nothing but a bundle of properties: properties have metaphysical priority over individuals and, therefore, they are the fundamental items of the ontology.

The assumption of an ontology of substances and properties is implicit in the quantum physicists' everyday discourse. Anchored in the ordinary language of subjects and predicates, they usually speak about electrons as having a certain momentum or photons as having a certain polarization, as if there existed an underlying "something" to which properties are "stuck". But perhaps the ordinary language is not the only factor that favors an ontological picture containing the categories of substance and of property. In the discourse of physics, states are what "label" the quantum systems and identify them; observables are "applied" to the states and are conceived as representing the properties of the system. In the orthodox formalism of quantum mechanics, the Hilbert space is taken as the basic formal element of the theory: states, represented by vectors of the Hilbert space, are logically prior; observables, in turn, are logically posterior since they are represented by operators acting on those previously defined vectors. When the logical priority of states over observables embodied in the Hilbert space formalism is endowed with an ontological content, the assumption of an ontology of substances and properties, with the traditional ontological priority of substances over properties, turns out to be "natural".

Our MHI, on the contrary, adopts an algebraic approach as its formal starting point. In this formalism, the basic element of the theory is the space of observables; states are logically posterior since they are represented by functionals over the space of observables. If this logical priority of observables over states is transferred to the ontological domain, the space of observables turns out to embody the representation of the elemental items of the ontology and the way in which they are arranged in a structure. On this basis, we introduce the following ontological interpretative postulates:

> **OIP1:** Given a quantum system represented by $\mathcal{S}\colon (\mathcal{O}, H)$, the observables $O \in \mathcal{O}$ ontologically represent *type-properties* $[O]$, and their corresponding eigenvalues o_i ontologically represent *case-properties* $[O\colon o_i]$ of the type-property $[O]$. In particular, the projectors $P \in \mathcal{O}$ are observables that ontologically represent type-properties $[P]$ with case-properties $[P\colon 1]$ and $[P\colon 0]$.

> **OIP2:** Given a composite quantum system represented by $\mathcal{S} = \mathcal{S}^1 \cup \mathcal{S}^2\colon (\mathcal{O}, H)$, where $\mathcal{S}^1\colon (\mathcal{O}^1, H^1)$ and $\mathcal{S}^2\colon (\mathcal{O}^2, H^2)$, and given the observables $A^1 \in \mathcal{O}^1$ of $\mathcal{S}^1$, $A^2 \in \mathcal{O}^2$ of $\mathcal{S}^2$, and the observables $A = A^1 \otimes I^2 \in \mathcal{O}$ and $A^f = f\left(A^1 \otimes I^2, I^1 \otimes A^2\right) \in \mathcal{O}$ of $\mathcal{S}$, where f is an analytical function, then, (i) the observables A and A^1 ontologically represent *the same type-property* $[A] = [A^1]$ with *the same case-properties* $[A\colon a_i^1] = [A^1\colon a_i^1]$, where the a_i^1 are the eigenvalues of both A and A^1, and (ii) the observable A^f ontologically represents a type-property $[A^f]$ with case-properties $[A^f\colon f(a_i^1, a_j^2)]$, where the a_i^1, a_j^2 are the eigenvalues of A^1 and A^2 respectively; $[A^f]$ is equivalent to the combination between $[A^1 \otimes I^2]$ and $[I^1 \otimes A^2]$, represented by the function f.

The interpretational postulate OIP2 expresses the usual quantum assumption according to which the observable A^1 of a subsystem $\mathcal{S}^1$ and the observable $A = A^1 \otimes I^2$ of the composite system $\mathcal{S} = \mathcal{S}^1 \cup \mathcal{S}^2$ represent the same property. On the other hand, this postulate establishes the necessary connections between the properties of the composite system and the properties

of its subsystems. The assumption of these connections is not a specific feature of quantum mechanics, but is also usual in classical mechanics where we consider, for instance, the energy of a two-particles composite system as a particular combination (expressed by the sum) of the energies of the component subsystems.

Summing up, whereas an ontology of substances and properties seems to be the natural reference of the theory in the Hilbert space formalism, the algebraic approach favors the assumption of an ontology of properties, where the ontological category of substance is absent.

7.2. POSSIBILITIES AND PROBABILITIES

Up to this point we have identified type-properties and case-properties in the ontology. However, these are not actual but possible properties.

The nature of possibility has been one of the most controversial issues in the history of philosophy. Nevertheless, two general conceptions can be identified, both of which find their roots in Antiquity. One of them, which is usually called "*actualism*", is the conception that reduces possibility to actuality. This was the position of Diodorus Cronus; in Cicero's words, "*Diodorus defines the possible as that which either is or will be*" (cited in Kneale and Kneale 1962, p. 117). This view survived over the centuries up to our time; for instance, for Bertrand Russell "possible" means "sometimes", whereas "necessary" means "always" (Russell 1919). The other conception, called "*possibilism*", conceives possibility as an ontologically irreducible feature of reality. From this perspective, the stoic Crissipus defined possible as "*that which is not prevented by anything from happening even if it does not happen*" (cited in Bunge 1977, p. 172). In present day metaphysics, the debate actualism-possibilism is still alive. For the actualists, the adjective "actual" is redundant: non-actual possible items (objects, properties, facts, etc.) do not exist, they are nothing. According to the possibilists, on the contrary, not every possible item is an actual item: possible items –*possibilia*– constitute a basic ontological category (see Menzel 2007).

As we have seen, according to modal interpretations, the formalism of quantum mechanics does not determine what actually is the case, but rather describes what *may* be the case with its corresponding probability. Once the actual-valued observables (type-properties) are selected by a certain rule of

actual-value-ascription, the actual occurrence of a particular value of such observables (a case-property) is an essentially indeterministic phenomenon which, as a consequence, cannot be determined by the theory. This means that, for each actual-valued observable, among all the possibilities described by the theory, only one is actually realized: the remaining possibilities do not become actual, and they might never become actual in the particular system under consideration. Nonetheless, from the realist perspective underlying modal interpretations, if quantum mechanics were true, it would describe reality. So, which is the reality accounted for by the theory? Certainly, not actual reality: if quantum mechanics is about what may be the case, it describes *possible reality.*

On this basis, according to our MHI quantum mechanics embodies a possibilist, non-actualist possibility: a possible property does not need to become actual to be real. This possibility is defined by the postulates of quantum mechanics and is not reducible to actuality. This means that reality spreads out in two realms, the *realm of possibility* and the *realm of actuality*. In Aristotelian terms, being can be said in different ways: as possible being or as actual being. And none of them is reducible to the other.

The non-actualist possibility is, then, conceived as an *ontological propensity to actualization*, whose measure is represented by the quantum probabilities and codified by the quantum state:

> **OIP3:** Given an elemental quantum system represented by $\mathcal{S}\colon(\mathcal{O}, H)$, its state $\rho \in \mathcal{O}'$ *codifies the ontological propensities to actualization* of the properties of $\mathcal{S}$, and the time evolution of ρ given by the Schrödinger equation ontologically represents *the time evolution of those ontological propensities.*

Since in our MHI probability is not defined by epistemic notions as evidence or hypothesis, the concept of probability is endowed with an ontological meaning. From our perspective probability is the measure of a possibilist, non-actualist possibility, whose real character does not depend on its actualization, and which applies to single quantum systems. As a consequence, the MHI does not favor a frequentist reading of probability, which is rooted in an actualist conception of probability and is unable to face the problem of the single-case probability assignment (see Giere 1976).

7.3. SYSTEMS AS BUNDLES OF POSSIBLE PROPERTIES

According to the traditional versions of the bundle theory, an individual is the convergence of certain case-properties, under the assumption that the type-properties corresponding to that individual are all determined in terms of an actual case-property. For instance, a particular billiard ball is the convergence of an actual value of position, an actual shape, say round, an actual color, say white, etc. So, in the debates about the metaphysical nature of individuals, the problem is to decide whether this individual is a substratum in which position, roundness and whiteness inhere, or it is the mere bundle of those case-properties. But in both cases the properties taken into account are actual properties. In other words, bundle theories identify individuals with bundles of *actual* properties.

The fact that our interpretation adopts an ontology of properties as the reference of quantum mechanics does not mean that it identifies the quantum system with a bundle of properties in the same sense as in traditional bundle theories, designed under the paradigm of classical individuals. We know that not all the possible type-properties lead to actual case-properties; only one of the case-properties of each type-property selected by the preferred context enters the realm of actuality. Of course, in each context one could insist on the classical idea of type-properties with their actual case-properties with no contradiction. In other words, the picture of a bundle of actual case-properties that defines a classical individual could be retained in each context. But as soon as we try to extend this ontological picture to all the contexts by conceiving the individual as a bundle of bundles, the Kochen-Specker theorem imposes an insurmountable barrier: it is not possible to actually ascribe the case-properties corresponding to all the type-properties to the system in a non-contradictory manner. Therefore, the classical idea of a bundle of bundles of *actual* properties does not work in the quantum ontology.

From our perspective, if the quantum ontology unfolds into two irreducible realms, the realm of possibility has to be taken into account when deciding what kind of properties constitutes the quantum bundle. In our interpretation, the quantum system is identified by its space of observables: its elements ontologically represent items belonging to the realm of possibility: the space of observables defines all the “possible” type-properties with their corresponding “possible” case-properties. Moreover, the realm of possibility is as real as the realm of actuality. From this viewpoint, it seems reasonable to

conceive a quantum system as the bundle of all the "possible" case-properties defined by the space of observables. This reading has the advantage of being immune to the challenge represented by the Kochen-Specker theorem, since this theorem imposes no restriction on possibilities. In other words, from our perspective the quantum system is not a bundle of actual case-properties as in the traditional bundle theories, but *a bundle of possible case-properties*: *it inhabits the realm of possibility*.

It is worth noting that, when the quantum system is conceived in this way, the account of its identity over time poses no difficulty: the space of observables remains invariant during the entire "life" of the system; the dynamics of the system is given only by the time evolution of propensities. On the other hand, nothing happening in the realm of actuality modifies the identity of the quantum system: it is the same no matter what possible case properties become actual.

7.4. THE GALILEAN INVARIANCE OF QUANTUM SYSTEMS

Any Galilean transformation T_α has to apply to the quantum system represented by $\mathcal{S}:(\mathcal{O}, H)$ as $\mathcal{S}: (\mathcal{O}, H) \rightarrow \mathcal{S}': (\mathcal{O}', H')$. However, since each T_α is an automorphism $T_\alpha : \mathcal{X} \rightarrow \mathcal{X}$, the Galilean transformations apply to the observables of the system in such a way that

$$\forall T_\alpha, \text{ if } O \in \mathcal{O} \text{ and } O \rightarrow O', \text{ then } O' \in \mathcal{O} \tag{7-1}$$

In other words, the space of observables of a quantum system is closed under the transformations of the Galilean group,

$$\forall T_\alpha, \ \mathcal{O} \rightarrow \mathcal{O}' \tag{7-2}$$

This feature is physically reasonable, since one does not expect that the mere application of a Galilean transformation on the system $\mathcal{S}$ modifies its identity by modifying its space of observables $\mathcal{O}$ (see Georgi 1982). Therefore, the result of the application of the Galilean transformations to a quantum system will only depend on the way in which the Hamiltonian is transformed:

$$\forall T_\alpha,\ \mathcal{S}:(\mathcal{O},H)\rightarrow\mathcal{S}':(\mathcal{O},H') \tag{7-3}$$

where H transforms unitarily as $H'=U_\alpha H U_\alpha^{-1}$, $U_\alpha=e^{iK_\alpha s_\alpha}$, and K_α is the generator of the transformation T_α.

As we have seen, when there are no external fields the Schrödinger equation is invariant under the Galilean group, and this means that the application of a Galilean transformation does not introduce a modification in the physical situation, but only expresses a change in the perspective from which the system is described. As a consequence, we can expect that, in this case, the system does not change its identity as the result of being Galilean-transformed: the system should be a Galilean-invariant object. In the context of the MHI, the invariance of the system under time-displacements, space-displacements and space-rotations follows directly from the invariance of the Hamiltonian under those transformations:

$$\mathcal{S}:(\mathcal{O},H)\rightarrow\mathcal{S}':(\mathcal{O},H')=\mathcal{S}:(\mathcal{O},H) \tag{7-4}$$

But the situation is, again, completely different for boost-transformations: although the Schrödinger equation is invariant, the Hamiltonian is not invariant under boosts.

Nevertheless, we have shown that any quantum system $\mathcal{S}$ with Hamiltonian $H=P^2/2m+W=K+W$ is a composite system $\mathcal{S}=\mathcal{S}_W\cup\mathcal{S}_K$, where $\mathcal{S}_W:(\mathcal{O}_W,H_W)$ is defined by the internal energy $H_W\in\mathcal{O}_W$, and $\mathcal{S}_K:(\mathcal{O}_K,H_K)$ is defined by the kinetic energy $H_K\in\mathcal{O}_K$, in such a way that

$$H=\frac{P^2}{2m}+W=H_K\otimes I_W+I_K\otimes H_W \tag{7-5}$$

Let us recall that, by OIP2, since $\mathcal{S}=\mathcal{S}_W\cup\mathcal{S}_K$, the observable $H_W\in\mathcal{O}_W$ of $\mathcal{S}_W$ and the observable $W=I_K\otimes H_W$ of $\mathcal{S}$ represent the same type property, $[H_W]=[W]$, with the same case-properties, $[H_W:w_\alpha]=[W:w_\alpha]$. We have also proved that, under boost transformations, the Hamiltonian transforms as

$$H \to H' = \left(\frac{P^2}{2m} + T_B \right) + W = H'_K \otimes I_W + I_K \otimes H_W \tag{7-6}$$

Therefore, the boost-transformed system is again a composite system $\mathcal{S} = \mathcal{S}_W \cup \mathcal{S}'_K$, whose elemental subsystems are the original $\mathcal{S}_W$ and the system $\mathcal{S}'_K : (\mathcal{O}_K, H'_K)$ now defined by a kinetic energy H'_K that adds the kinetic energy H_B of the boost to the original kinetic energy H_K.

This argument shows that, when there are no external fields, a boost-transformation acts on a system represented by $\mathcal{S} = \mathcal{S}_W \cup \mathcal{S}_K$ as

$$\mathcal{S} = \mathcal{S}_W \cup \mathcal{S}_K \to \mathcal{S}' = \mathcal{S}_W \cup \mathcal{S}'_K \tag{7-7}$$

When, in particular, $\mathcal{S}$ is described in the reference frame at rest with respect to its center of mass, $P = 0$; then, $\mathcal{S}$ is an elemental system with Hamiltonian $H = W$, on which a boost acts as

$$\mathcal{S} = \mathcal{S}_W \to \mathcal{S}' = \mathcal{S}_W \cup \mathcal{S}'_K \tag{7-8}$$

where the subsystem $\mathcal{S}'_K$ is now defined only by the kinetic energy of the boost. Therefore, the subsystem $\mathcal{S}_W$, carrying the internal energy of the system, is boost-invariant, in agreement with the fact that the internal energy W is a Casimir operator of the Galilean group. The application of a boost-transformation only affects the subsystem $\mathcal{S}_K$ by adding the kinetic energy of the boost to its Hamiltonian:

$$\mathcal{S}_W \to \mathcal{S}'_W = \mathcal{S}_W \qquad H'_W = H_W \tag{7-9}$$

$$\mathcal{S}_K \to \mathcal{S}'_K \qquad H'_K = H_K + H_B \tag{7-10}$$

This result leads us to ask ourselves about the ontological status of both subsystems.

On the one hand, when there are no external fields, the action of a boost-transformation has a well-defined manifestation in the energy spectrum of the composite system $\mathcal{S} = \mathcal{S}_W \cup \mathcal{S}_K$: the boost produces a Doppler shift on the energy of $\mathcal{S}$. But we also know that energy is defined up to a constant value:

the relevant information about the energy spectrum of a system is contained in its internal energy, and the kinetic energy only introduces a shift of that spectrum. Therefore, the boost-invariant subsystem $\mathcal{S}_W$ carries the physically meaningful structure of the energy spectrum, and $\mathcal{S}_K$ represents an energy shift which, although observable, is physically non relevant and merely relative to the reference frame used for the description. On the other hand, even the composite or elemental character of the system $\mathcal{S}$ depends on the particular reference frame selected. In fact, in the reference frame RF at rest with respect to the center of mass, $\mathcal{S} = \mathcal{S}_W$ is an elemental system; when, in turn, we decide to describe the system in a reference frame RF' uniformly moving with respect to RF, the system turns out to be composite, $\mathcal{S} = \mathcal{S}_W \cup \mathcal{S}_K$.

Both considerations point to the same direction: the *objective* content of the description is given by the internal energy. In other words, the objective description of a system is $\mathcal{S}_W$, that is, the description in the reference frame at rest with respect to the center of mass, where $H = W$: $\mathcal{S}_W$ is completely invariant under the Galilean group. On the contrary, $\mathcal{S}_K$, which carries the kinetic energy, is a sort of "pseudo-system", whose identity is modified by a mere change of the descriptive perspective, and may even "appear" and "disappear" as a consequence of such a change. On this basis, we can say that $\mathcal{S} = \mathcal{S}_W \cup \mathcal{S}_K$ and $\mathcal{S}_W$ refer to the same *ontological system*:

$$[\mathcal{S}] = [\mathcal{S}_W \cup \mathcal{S}_K] = [\mathcal{S}_W] \tag{7-11}$$

where the symbol '=' strictly denotes *logical identity* (that is, if $a = b$, then a and b are two names for the same item). Therefore, when ontological systems are free from external fields, they are invariant under all the transformations of the Galilean group, in particular, under boosts,

$$\begin{aligned} &[\mathcal{S}] = [\mathcal{S}_W \cup \mathcal{S}_K] = [\mathcal{S}_W] \rightarrow \\ &\quad \rightarrow [\mathcal{S}'] = [\mathcal{S}_W \cup \mathcal{S}'_K] = [\mathcal{S}_W] = [\mathcal{S}] \end{aligned} \tag{7-12}$$

The intuition about a strong link between invariance and objectivity is rooted in a natural idea: what is objective should not depend on the particular perspective used for the description; or, in group-theoretical terms, what is

objective according to a theory is what is invariant under the symmetry group of the theory. This idea is not new. It was widely discussed in the context of special and general relativity with respect to the ontological status of space and time: "*Henceforth space for itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality*" (Minkowski, 1923, p. 75). The claim that objectivity means invariance is also a central thesis of Weyl's book *Symmetry* (1952). In recent times, the idea has strongly reappeared in several works. For instance, in her deep analysis of quantum field theory, Auyang (1995) makes her general concept of "object" to be founded on its invariance under transformations among all representations. In turn, the assumption that invariance is the root of objectivity is the central theme of Nozick's book *Invariances: The Structure of the Objective World* (2001). Our conclusion about the objective description of a quantum system is in complete agreement with the general idea behind those works: when the Galilean group leaves invariant the Schrödinger equation, the objective description of the system is also invariant and, as a consequence, the ontological system is left unaffected by the Galilean transformations.

Chapter 8

CONCLUSIONS

In this work we have presented a new realist, non-collapse interpretation of quantum mechanics, which moves away from the prevailing trend in the subject by paying special attention to the physical relevance of the interpretation. In particular, our proposal endows the Hamiltonian of the system, systematically ignored in the traditional interpretations, with a central role: it distinguishes between systems and subsystems, and is the main ingredient in the selection of the actual-valued observables. The main advantages of the MHI are the following:

- The interpretation is Galilean-invariant: as expected, the preferred context does not depend on the reference frame selected for the description.
- The application of the interpretation to several physical situations shows its agreement with theoretical commitments and empirical evidence coming from the practice of physics.
- When used to account for quantum measurements, the interpretation not only explains the definite reading of the pointer both in the ideal and in the non-ideal case, but also accounts for the difference between reliable and non-reliable measurements, in accordance with experimental practice
- The interpretation is compatible with the decoherence approach in the sense that the preferred context defined by the Actualization Rule agrees with the pointer basis selected by the environment-induced decoherence.
- The interpretation describes the elemental categories of the ontology referred to by quantum mechanics: an ontology with two irreducible and

> equally real realms, the realm of actuality and the realm of possibility, where quantum systems are bundles of possible properties

Of course, the MHI has not supplied solutions to all the challenges raised by quantum mechanics. Nevertheless, on the basis of its advantages, it deserves to be considered for further developments.

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